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Forecasting non-linear macroeconomic indexes of India: an ensemble of MLP and Holt's linear methods

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Abstract
Electricity and electric-equipment play a critical role in modern living, and precise price forecasting for these items' aids decision-makers in anticipating changes, planning, and budgeting ahead of time. This present research focused on the wholesale price indexes of items from the "manufacture of electrical equipment (MEEQ)" group of India's existing wholesale price index (WPI) series. This work proposed a novel, state-of-art ensemble forecast approach that used multilayer perceptron (MLP) and Holt's linear (HL) approaches for some specified WPIs from this group. The researchers selected the WPIs that manifest non-linearity as determined by the curve-fit technique. The paper applied a variance-based weighted average scheme to generate the ensemble forecast. The statistical rigor-based curve-fit aided in identifying that seventeen out of forty-eight indexes manifest non-linear fits. The proposed MLP-HL ensemble approach exhibited excellent forecast results (relative root mean squared error, i.e., RRMSE < 10%) for all seventeen WPIs. For each of these seventeen WPI's, the current work compared the proposed MLP-HL with nineteen models (eight statistical, four machine learning, and seven contemporary ensemble strategies). In the majority of the cases, it outran the nineteen models in terms of mean absolute error (MAE) and the mean absolute percentage error (MAPE). As an alternative pathway for forecasting these seventeen non-linear WPIs, the present work suggests using this proposed MLP-HL approach.

Keywords
Curve-fit, Multilayer perceptron, Holt's linear, Ensemble forecasting, Wholesale price index.

1.Introduction
People use different indices as indicators of the economy and employ them to review the economic condition. They utilize the proficiency accumulated from studying and evaluating these indexes to govern a country's economy. In India, the wholesale price index (WPI) - a macroeconomic indicator, is used by diverse sections - such as government, economists, statisticians, researchers, industries, business houses. It (India's existing WPI, base 2011-12=100) lists the WPIs of five hundred sixty-four individual manufactured items/products, arranged into twenty-two groups, e.g., food products, textiles, electrical equipment, electronic products, machinery, motor vehicle [1].

The importance of electricity and electrical equipment in modern life is paramount. The industry and modern world, starting from Industry 2.0 to date, heavily depends on electric power and various electrical equipment. Price change in these types of equipment impacts the price of many other manufactured products as the manufactured products are the result of complex business processes.

Therefore, accurate price forecasting of these equipments helps the decision-makers to anticipate changes, plan, and budget proactively. India's WPI lists forty-eight types of equipment under the manufacture of electrical equipment (MEEQ) group [1].

The path and advancement of the time-series (TS) forecast are indeed evergreen and full of challenges. A primary goal of this field (TS-forecasting) is to furnish a reliable forecast by mining the inherent structure and hidden information of the TS and applying it to the model appropriately. Recent works suggest that in TS-forecasting, there exist many approaches, e.g., exponential smoothing (ES) [2, 3], auto-regressive integrated moving average (ARIMA) [4, 5] artificial neural network (ANN) [6, 7], extreme learning machine (ELM) [8, 9], Facebook-prophet

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(FB-Prophet) [10, 11], support vector regression (SVR) [12, 13]. Researchers have also applied ensemble approaches for TS-forecasting and realized effective forecasting, as apparent from the pieces of literature [14−16]. It is also observed from the contemporary works that researchers have forecast several indices, e.g., stock index [17, 18], consumer price index (CPI) [19, 20], WPI [21, 22] using diverse techniques.

1.1Motivation

None of the approaches is an unambiguous winner in TS-forecasting. It's a challenge to present a convenient alternate forecasting system that is straightforward to implement and provides high forecast accuracy. In addition, prediction of future values through any single approach may be counterproductive due to inaccuracy and complicatedness. These challenges inspire the authors in this paper to put together an ensemble of a multilayer perceptron (MLP) and Holt's linear (HL) approaches to design and develop the forecasting model for the WPIs from the MEEQ group of India's WPI-series that show non-linear fit (NonLinear_WPIs).

1.2Objectives of the study

The authors set the objectives of the present work that are as follows:

- To identify the individual item's WPI from the MEEQ group of India's current WPI-series that display the non-linear fit/trend where the authors utilized the curve-fit technique to identify trends.
- To build an ensemble forecast approach efficient to produce twelve months ahead values by utilizing the forecasts of MLP and HL techniques for the NonLinear_WPIs.
- To identify the ability of the offered ensemble approach in furnishing forecasts for the selected indexes and quantify the forecasting grades (excellent/ very good/ fair/ poor).
- To contrast, the out-of-sample forecast efficiency of the offered ensemble approach with different statistical, machine learning (ML), and ensemble approaches using multiple performance metrics.

The remaining sections are organized as follows: discussions about the previous works of literature and its findings (literature review section); methodology of the work that includes research outline, data description, trend analysis, proposed ensemble approach, experimental setup, and applied forecast accuracy metrics in the methodology section; the result section discusses and exhibits the data analysis and findings; the discussion section contains the detailed discussions about the significant findings of the work and the limitations; the last section includes the conclusive remarks and future scope.

2.Literature review

The curve-fit process constructs a curve that best-fit or has the close vicinity to the dataset, helps in illuminating the trends underneath the data [23, 24], and forecasting coming values [25–29]. The studies have shown the usage of this method to forecast prospective coming values in the stock market [25, 26] and health informatics [27, 28].

Recent studies have observed that the popular ARIMA is used to forecast various economic as well as financial TS data successfully - such as the CPI of India [30], Jordan [19], Somaliland [20], Germany [31]; the stock index of Nigeria [17], India [18]; prices of potato [32, 33], onion in India [32].

In [34], the authors have applied single, double, and Holt-Winters (HW) ES techniques to examine the trends of Nigerian temperature data. The authors have employed HL and the Double ES techniques for forecasting the stock market prices of crude oil and some metals - Gold, Silver, Platinum [35]. In [36], the HL has exhibited better performance than ARIMA to forecast Kijang Emas's price. The researchers have adopted the error, trend, seasonal (ETS) model - a state-space model, a class of ES techniques for different TS data forecasting, such as air passengers in Kuwait [37], COVID19 in India [38], the stock price [39].

In recent past studies, several authors have applied the ANN technique to forecast different TS data. They have employed the ANN technique to predict the future values of peak load hours in Russia [40] and Iranian wheat demand [41]. The MLP has efficiently forecasted the monthly price of copper [42], monthly bacille Calmette-Guerin (BCG) vaccine demands in the Philippines [43], and Ecuadorian textile product demand [44].

In [8], the authors have forecasted the price of Arabica and Robusta coffee using ELM, MLP, ES, Autoregressive (AR), and ARIMA approaches and noted that the ELM's accuracy is better. In [9], the authors have observed the ELM and MLP exhibited better accuracy than ES, ARIMA, drift, average, and naive approaches to forecast the bitcoin price. In [45], the authors have contrasted the performances of wavelet neural networks, backpropagation algorithm, and ELM in predicting two non-linear TS data (i.e., prices of soybean sack and demands of food products). In recent studies, the authors have utilized SVR for forecasting multifarious TS data - e.g., ship price [46], electric load [47], and the stock price [48]. In [46], the authors have used this technique for shipping business forecasting, and the SVR performed superior to ARIMA. In [47], the researchers have developed a new SVR and a basic SVR strategy for forecasting the electric load in Australia and analyzed their performances. The researcher has compared ARIMA, neural networks, and SVR for predicting the various stock indexes - e.g., NSE, BSE, S &P500 [48].

The authors have employed seasonal Naive, regressions, ARIMA, ETS, and FB-Prophet in forecasting daily COVID-19 cases [49]. In [10], the authors have compared additive, ARIMA, and FB-Prophet approaches in forecasting supermarket sales data and noted the most suitability of the FB-Prophet. For forecasting the CPI of Ecuador, the researchers have employed several approaches - e.g., neural networks, SVR, ES, FB-Prophet [50].

The application of ensemble approaches for TS-forecasting is also observed in recent works. For forecasting the electricity consumption of Spain, the authors have combined the following three approaches to develop an ensemble approach - decision tree, random forest, and gradient boosted trees [51]. In [52], to predict the non-linear Baltic dry index, the authors have devised a heterogeneous ensemble approach by employing multi-objective PSO and a mutation operator of the dynamic heterogeneous type. The authors in [53] have proposed an ensemble of two deep learning approaches - Bi-directional and Convolutional long short term memory (LSTM) to forecast COVID19. To predict three non-linear stock indexes - Dow Jones, S&P500, and NASDAQ, the authors have proposed an ensemble approach that employed Gaussian process regression, recurrent neural network (RNN), and LSTM [54].

In [55], for forecasting the S&P500 and CSI300, the authors have combined LSTM with complete ensemble empirical mode decomposition with adaptive noise (CEEMDAN) - a type of empirical mode decomposition technique. For price forecasting of wheat and soybean in Brazil, the authors have noted an ensemble of the extreme gradient boosting machine or STACK and random forest provided the best performance [56].

2.1Findings of the literature review & research gap identification

Observations from the works of literature are as follows:

- Different researchers have employed the curve-fit technique to identify the trends of TS data and for forecasting TS data.
- For forecasting TS data, the researchers have frequently employed ARIMA and various ES methods. The approaches yielded effective outcomes.
- Different researchers have used diverse ML methods - e.g., neural approach, SVR, and FB-Prophet and got good results in the cases of TS forecasting.
- Different researchers have effectively employed several ensemble strategies for forecasting TS data.

Research gaps as noted from the recent works in the context of the WPIs from the MEEQ group of India's WPI-series: building forecast approaches by employing regressions, ARIMA, HW, ETS, simple exponential smoothing (SES), MLP, ELM, FB-Prophet, SVR, and ensemble techniques for the indexes that show non-linear fit.

For bridging these research gaps, the paper proposes an ensemble forecast approach by utilizing the forecasts of MLP and HL techniques (MLP-HL) for the NonLinear_WPIs and compares its forecast performance with regressions, ARIMA, HW, ETS, SES, MLP, ELM, FB-Prophet, SVR, and other ensemble approaches.

3.Methodology

The following illustration (*Figure 1*) outlines the schematic sketch of the methodology adopted by the researchers. The methodology adopted has the following steps: (i) Data collection and partition, (ii) Identification of non-linear trends, (iii) Development of proposed MLP-HL ensemble model, (iv) Performance evaluation of the proposed MLP-HL model employing relative root mean squared error (RRMSE) [57], (v) Development of other models, and (vi) Comparing the forecast performance of the offered MLP-HL model with others employing mean absolute percentage error (MAPE) [58], mean absolute error (MAE) [58], and root mean squared error (RMSE) [58].

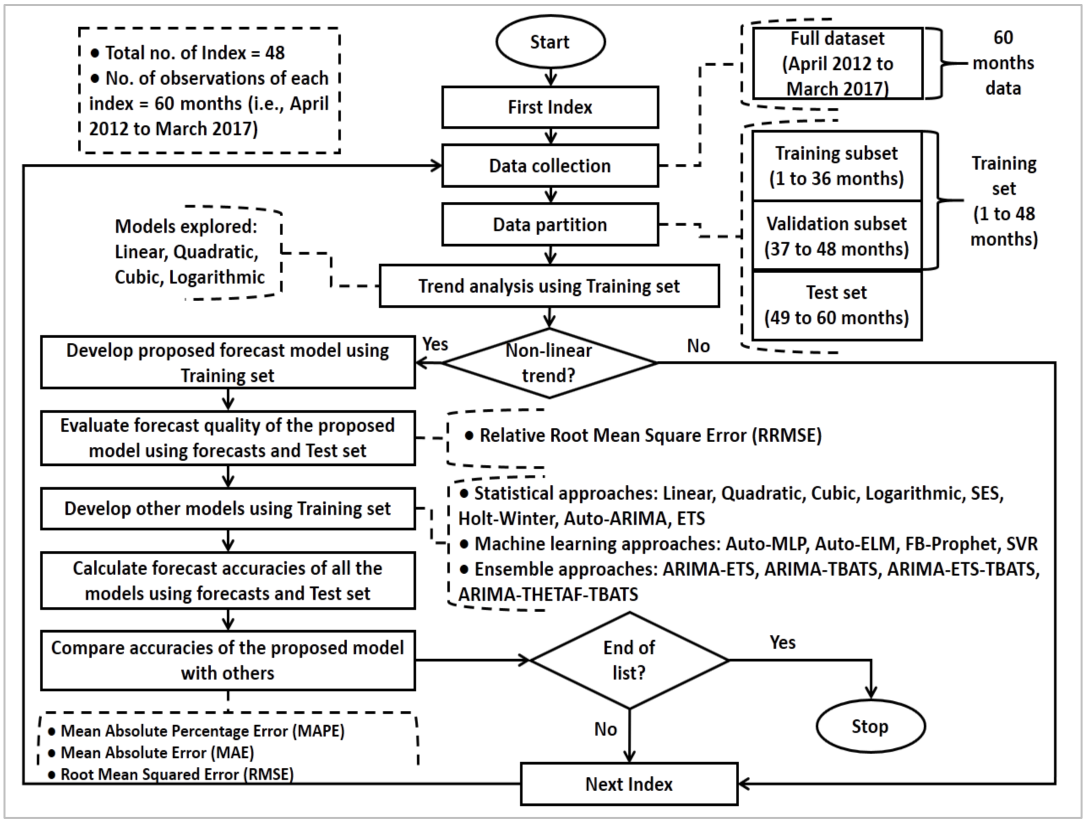

Figure 1 Schematic sketch of the adopted methodology

3.1Data collection and partition

For this work, the researchers looked at forty-eight individual items from India's current WPI-series' MEEQ group, collected their monthly data up to March 2017 from April 2012 - a total of sixty months of data for each index [1]. *Table 1* enumerates the split-up of each index's dataset used for training, validation, and testing.

Table 1 Split-up of each WPI's dataset

S. No.	Starting from	Ending in	Duration	Description
1.	April 2012	March 2017	60 months	Full dataset for each WPI
2.	April 2012	March 2015	36 months	Training subset – used to tune the MLP's hyper-parameter
3.	April 2015	March 2016	12 months	Validation subset – used to tune the MLP's hyper-parameter
4.	April 2012	March 2016	48 months	Training set – used to develop models
5.	April 2016	March 2017	12 months	Test set – used to calculate the forecast efficiencies of the models

3.2Trend analysis using curve-fit technique

This paper used the curve-fit technique to examine the trends of the forty-eight WPIs and employed the linear, quadratic, cubic, and logarithmic regression models [23, 26]. For each WPI, the authors in this paper identified the models with $R^2 \geq .7$, Adjusted $R^2 \geq .7$, and Significance of F test < .05 as the competing models, and the one with the lowest Akaike Information criteria (AIC) considered as the probable best-fit amongst them. This identified model was subjected to residual diagnostics (Box plot to test outliers and Normal QQ plot to test normal distribution) to recognize the best fit. This paper applied the lm (stats package) [59], qqnorm (stats package) [59], and boxplot (graphics package) [59] of R for model building, testing normal distribution and the presence of outliers, respectively. *Figure 2* illustrates the schematic outline of the trend analysis employing the curve-fit.

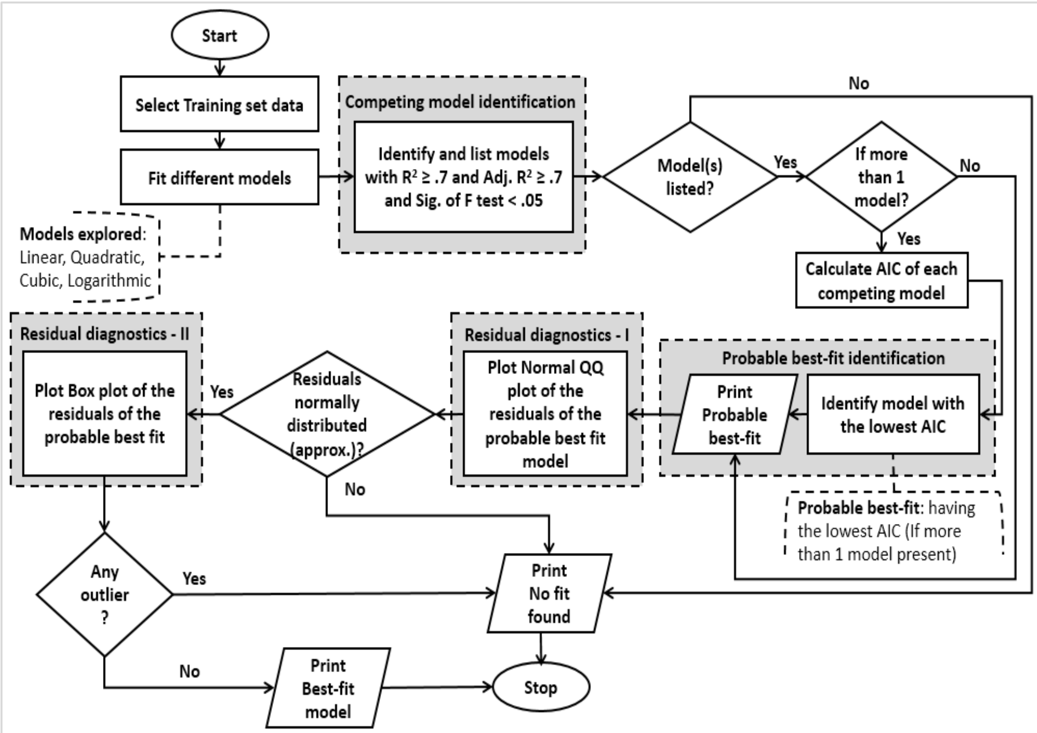

Figure 2 Trend analysis of each index using best-fit curve identification

3.3Proposed MLP-HL ensemble model development

Currently, this work adopted the MLP and HL techniques to create the ensemble model for forecasting. The ANN is good at learning and modeling data that have non-linearity [60, 61]. The researchers applied the MLP, a class of neural approach, in the present work. To forecast the univariate TS data with a trend, the HL is a commonly used technique, and the forecast values either constantly increase or decrease and tend to over forecast [62]. In the offered approach, the researchers combined both MLP and HL - as combining diverse methodologies can be an effective strategy to enhance forecast results.

The proposed ensemble strategy employed two layers wherein the bottom layer exercised two approaches - the HL and MLP to produce top-layer inputs. In the top layer, the strategy exercised the variance-based weighted average technique to produce the final forecasts. This work applied Forecast_comb (ForecastCombinations package) of R for the ensembling [63].

Figure 3 demonstrates the ensembling strategy applied on the non-linear trend exhibiting WPIs to forecast twelve months' advance values in each case.

The employed MLP approach consists of one input layer, two hidden layers, and one output layer. Here, the researchers applied the lag1 and lag2 of the TS data as inputs. The authors identified the best combination of the numbers of neurons in the first and second hidden layers by the hyper-parameter tuning from the search space. The hyper-parameter search spaces are defined as: (a) in the first hidden layer, the no. of neuron (N1) = {1, 2}, (b) in the second hidden layer, the no. of neuron (N2) = {1, 2, ..., 5}. The hyper-parameter search space can be defined in set notation as Equation 1:

$$S = \{(a, b)\ \forall\ a \in N1, b \in N2\} \quad (1)$$

The current work operated the following for materializing the MLP:

- Algorithm - 'Resilient back propagation with weight backtracking'
- Activation function - 'logistics'

The final forecast is an ensemble of twenty trained networks obtained using a kernel density estimation-based mode operator.

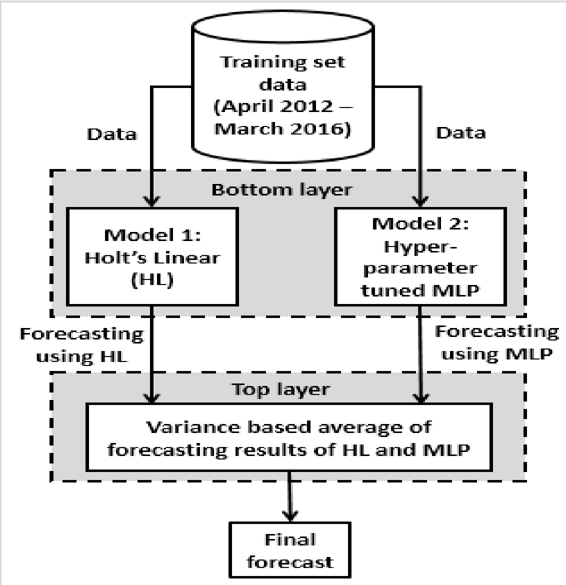

Figure 3 Ensemble MLP-HL model development

Figure 4 exhibits the hyper-parameter tuning strategy of the component MLP model.

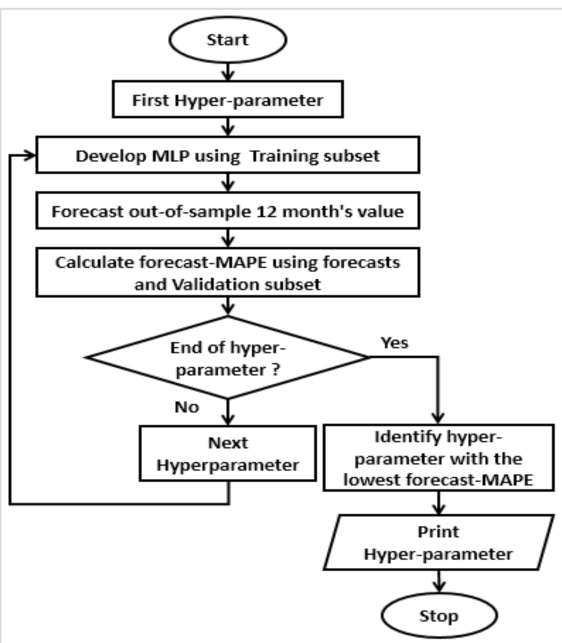

Figure 4 Hyper-parameter tuning flowchart of the MLP

3.4Other forecast approaches

This research employed nineteen different strategies for forecasting the NonLinear_WPIs and conducted a performance contrast of them with the offered MLP-HL model. The research utilized the following statistical methodologies for forecasting: Regression models (Linear, Logarithmic, Quadratic, and Cubic), ES models (SES, HW, and ETS), and the Auto ARIMA model. It further employed four ML approaches for forecasting - Auto MLP, Auto ELM, FB-Prophet, and SVR. This research also adopted seven contemporary ensemble strategies for forecasting that are: (i) ARIMA-ETS using CV errors weight (En1) [64, 65], (ii) ARIMA-ETS using equal weight (En2) [65, 66], (iii) ARIMA-TBATS applying equal weights (En3) [65, 67], (iv) ARIMA-TBATS applying CV errors weights (En4) [65], (v) ARIMA-ETS-TBATS using equal weights (En5) [65, 67], (vi) ARIMA-ETS-TBATS employing CV errors weights (En6) [65], and (vii) ARIMA-THETAF-TBATS using equal weights (En7) [68].

3.5 Experimental setup

This research collected data for forty-eight indices from the MEEQ group. The data features are as follows: granularity monthly, length sixty months, and no missing value. The researchers split the data into the training set (one to forty-eight months' data) and the test set (forty-nine to sixty months' data). They used the training set to identify the non-linear indices (employing curve-fit) and develop forecast models. They computed forecast accuracies using the Test set. To model, this research utilized the following packages from the R software: Regression models – stats [59]; ES models and Auto ARIMA model – forecast [69, 70]; MLP and Auto ELM models – nnfor [71]; FB-Prophet model – prophet [72]; SVR model - e1071 [73]; En1, En2, En3, En4, En5, En6, and En7 models – forecastHybrid [74].

3.6 Forecast accuracy metrics

This research employed the RRMSE to evaluate the performance of the MLP-HL ensemble approach. If the RRMSE was less than ten percent, the authors judged the model accuracy to be excellent [57]. To conduct a performance comparison of the MLP-HL model with others, the authors employed the following two accuracy metrics: MAPE [58], MAE [58], and RMSE [58].

4.Results

4.1Results of trend analysis of forty-eight indexes from the MEEQ Group of India's WPI-series

In this research, the researchers analyzed the trends of forty-eight WPIs and arranges the findings in *Table 2* and *Table 3*. Eighteen out of forty-eight WPIs, i.e., thirty-seven point five percent exhibit fits (*Table 2*). Seventeen out of eighteen fit-exhibiting WPIs (approximately ninety-four percent) are non-linear fits, where the majority are cubic (*Table 3*).

The paper arranges the list of WPIs with non-linear fits in *Table 4*.

Table 5 presents the trend, linearity, and curvature attributes of the non-linear WPIs. The non-linear indices exhibited heterogeneity.

Table 2 Fits found versus no fit found

Category	Number of index	Percentage of index
No fit found	30	62.50
Fit found	18	37.50
Total	48	100.00

Table 3 Analysis of different types of fit found

Category	Number of index	Percentage of index
Linear	1	5.56
Quadratic	6	33.33
Cubic	11	61.11
Logarithmic	0	0
Total	18	100.00

Table 4 WPIs from MEEQ group in India showing non-linear trends

Sl. No.	WPI	Code used	Best fit
1.	"Cooling tower"	WPI1	Quadratic
2.	"Rotor/magneto rotor assembly"	WPI2	Quadratic
3.	"Electric switch gear control/ starter"	WPI3	Cubic
4.	"Meter Panel"	WPI4	Cubic
5.	"Amplifier"	WPI5	Cubic
6.	"Multimeter"	WPI6	Quadratic
7.	"Batteries"	WPI7	Cubic
8.	"Fibre Optic Cables"	WPI8	Quadratic
9.	"PVC Insulated Cable"	WPI9	Cubic
10.	"Jelly Filled Cables"	WPI10	Cubic
11.	"Aluminium/Alloy Conductor"	WPI11	Cubic
12.	"Aluminium wire"	WPI12	Cubic
13.	"Light fitting accessories"	WPI13	Cubic
14.	"Refrigerators"	WPI14	Cubic
15.	"Fan"	WPI15	Quadratic
16.	"Washing Machines/Laundry Machines"	WPI16	Quadratic
17.	"Electric welding machine"	WPI17	Cubic

Table 5 Features of the non-linear WPIs

S. No.	WPI	Trend	Linearity	Curvature
1.	WPI1	0.920	6.154	-0.837
2.	WPI2	0.960	-6.701	-0.558
3.	WPI3	0.956	6.219	-1.863
4.	WPI4	0.962	5.628	3.006
5.	WPI5	0.903	5.873	2.219
6.	WPI6	0.990	6.721	1.640
7.	WPI7	0.972	6.778	-1.189
8.	WPI8	0.928	4.725	-3.967
9.	WPI9	0.900	3.599	-3.175
10.	WPI10	0.814	3.137	-3.770
11.	WPI11	0.883	4.961	-2.612
12.	WPI12	0.865	2.827	-4.391
13.	WPI13	0.917	5.907	0.771
14.	WPI14	0.959	5.985	-1.417
15.	WPI15	0.923	6.401	-1.021
16.	WPI16	0.773	5.379	-2.050
17.	WPI17	0.840	-4.476	-3.296

4.2Hyper-parameter tuning of MLP for building the proposed MLP-HL ensemble forecast model

The researchers in this work tuned the MLP (component model of MLP-HL) hyper-parameter, considering the hyper-parameters from the provided search space, and obtained the optimized MLP for every WPI. The paper arranges the optimized MLP architecture for each seventeen non-linear WPIs in *Table 6*.

Table 6 Optimized MLP models of the seventeen non-linear WPIs

S. No.	WPI	Optimized MLP model
1.	WPI1	2/1/2/1
2.	WPI2	2/2/3/1
3.	WPI3	2/2/2/1
4.	WPI4	2/2/5/1
5.	WPI5	2/1/5/1
6.	WPI6	2/2/3/1
7.	WPI7	2/2/1/1
8.	WPI8	2/1/2/1
9.	WPI9	2/2/1/1
10.	WPI10	2/2/1/1
11.	WPI11	2/1/3/1
12.	WPI12	2/2/2/1
13.	WPI13	2/2/5/1
14.	WPI14	2/1/3/1
15.	WPI15	2/2/2/1
16.	WPI16	2/2/2/1
17.	WPI17	2/2/2/1

4.3Forecasting & performance evaluation of the proposed MLP-HL ensemble model

In this work, for each NonLinear_WPIs, the authors applied the offered MLP-HL strategy, produced twelve months forward (ahead) predictions, and calculated the forecast accuracies. *Figure 5* depicts the visualization of the forecasting, and *Table 7* tabulates the RRMSE values. *Table 7* shows that the proposed MLP-HL ensemble approach exhibited excellent forecast performances for all the seventeen WPIs (i.e., RRMSE in each case < 10).

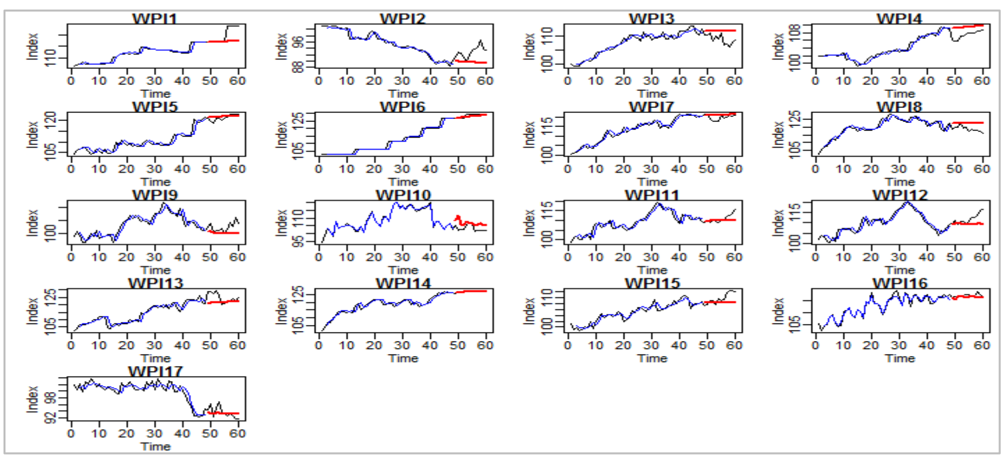

Figure 5 Visualization of observed, predicted, and twelve step out-of-sample forecasts using proposed MLP-HL ensemble model of the seventeen non-linear WPIs; observed - black line, in-sample prediction - blue line, and out-of-sample forecasts - red line

Table 7 Forecast accuracies of the proposed MLP-HL ensemble model of the seventeen non-linear WPIs

S. No.	WPI	RRMSE (12 months ahead)
1.	WPI1	3.53
2.	WPI2	4.35

S. No.	WPI	RRMSE (12 months ahead)
3.	WPI3	3.75
4.	WPI4	1.96
5.	WPI5	0.93
6.	WPI6	0.84
7.	WPI7	1.95
8.	WPI8	4.37
9.	WPI9	2.62
10.	WPI10	2.35
11.	WPI11	1.45
12.	WPI12	3.16
13.	WPI13	3.58
14.	WPI14	0.65
15.	WPI15	1.62
16.	WPI16	1.73
17.	WPI17	1.70

Table 8 represents the correlation coefficient and coefficient of determination (R2) of the proposed MLP-HL. The MLP-HL exhibited substantial R2 (> .75) for the fifteen and moderate R2 (> 0.5) for the two indices (i.e., WPI10 and WPI16). It indicated a better fit for fifteen indices, and for two indexes the fit is reasonable.

Table 8 Statistical measure of the MLP-HL model

S. No.	WPI	Correlation coefficient	Coefficient of determination
1.	WPI1	0.95	0.91
2.	WPI2	0.97	0.95
3.	WPI3	0.96	0.92
4.	WPI4	0.98	0.95
5.	WPI5	0.93	0.86
6.	WPI6	0.98	0.96
7.	WPI7	0.97	0.94
8.	WPI8	0.94	0.88
9.	WPI9	0.92	0.84
10.	WPI10	0.83	0.69
11.	WPI11	0.92	0.85
12.	WPI12	0.94	0.88
13.	WPI13	0.94	0.88
14.	WPI14	0.95	0.90
15.	WPI15	0.92	0.84
16.	WPI16	0.85	0.73
17.	WPI17	0.89	0.80

Table 9 represents the 'area under ROC curve' (AUC) score of the MLP-HL. The AUC score was excellent for fifteen indices (0.9-1), and for two indices (i.e., WPI14 and WPI17), it was good (0.8-0.9). It showed that the prediction (in-sample) ability of the MLP-HL is high.

Table 9 AUC score of the MLP-HL model

S. No.	WPI	AUC score
1.	WPI1	0.99
2.	WPI2	1.00
3.	WPI3	1.00
4.	WPI4	1.00
5.	WPI5	0.95
6.	WPI6	1.00
7.	WPI7	0.99
8.	WPI8	0.93

S. No.	WPI	AUC score
9.	WPI9	0.97
10.	WPI10	0.91
11.	WPI11	0.98
12.	WPI12	0.96
13.	WPI13	0.98
14.	WPI14	0.88
15.	WPI15	0.98
16.	WPI16	0.90
17.	WPI17	0.81

4.4 Performance comparison of the proposed MLP-HL ensemble with others

In this research, for each WPI, the researchers performed twelve months forward (ahead) forecasting employing the proposed MLP-HL approach, the component models (optimized MLP and HL), and nineteen selected models (eight statistical, four ML, and seven ensembles). The researchers calculated the MAPE and MAE of each model for each WPI. They compared the MAPE and MAE of the proposed MLP-HL approach with the MAPE and MAE of these twenty-one models (two component-model and nineteen selected models) for each WPI. They calculated the percentage of cases when the proposed MLP-HL approach performed better than the others, i.e., the accuracies of the MLP-HL approach were less than the other. *Table 10* enumerates the findings of the performance of the MLP-HL and component models. For the majority, the twelve-month forward forecast-MAPE and forecast-MAE of the MLP-HL were less than the component models, i.e., the forecast-performance of the suggested MLP-HL was better.

Table 10 Performance comparison of the MLP-HL ensemble with component models

Accuracy metrics	Criteria	Criteria match
Forecast-MAPE	MLP-HL_{MAPE} < Optimized MLP_{MAPE}	For 64.71% of WPIs
	MLP-HL_{MAPE} < HL_{MAPE}	For 58.82% of WPIs
Forecast-MAE	MLP-HL_{MAE} < Optimized MLP_{MAE}	For 64.71% of WPIs
	MLP-HL_{MAE} < HL_{MAE}	For 58.82% of WPIs

Figure 6, *Figure 7* and *Figure 8* depict the performance comparison of MLP-HL and twelve models (eight statistical and four ML) in terms of forecast-MAPE, forecast-MAE, and forecast-RMSE, respectively. For the bulk of the WPIs, the twelve-month forward forecast accuracies of the suggested MLP-HL were smaller than the eleven out of twelve models (except SES), i.e., the proposed MLP-HL performed better than these eleven models as per all three accuracy metrics. The MLP-HL performed better than the SES as per forecast-MAPE (in 52.94% cases) and forecast-MAE (in 52.94% cases).

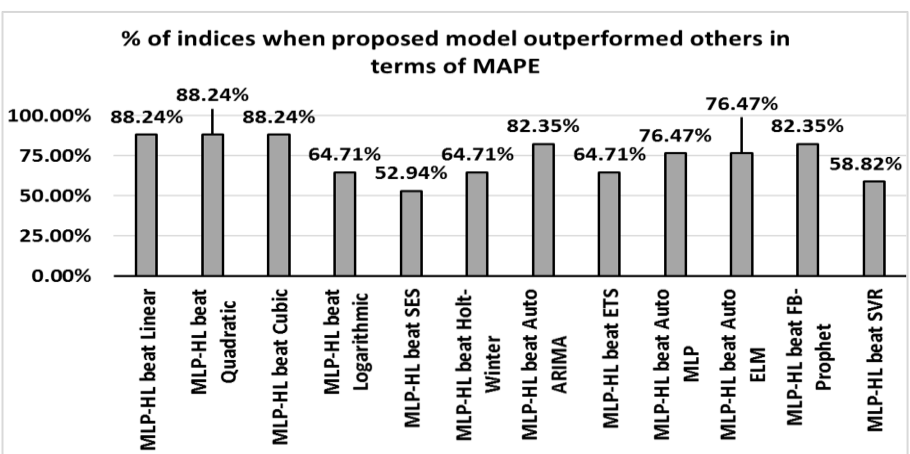

Figure 6 Forecast-MAPE comparison of proposed MLP-HL with twelve different models

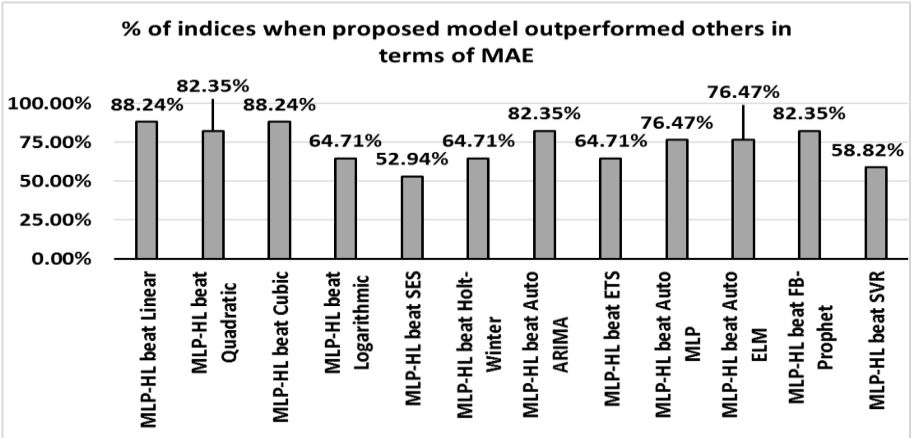

Figure 7 Forecast-MAE comparison of proposed MLP-HL with twelve different models

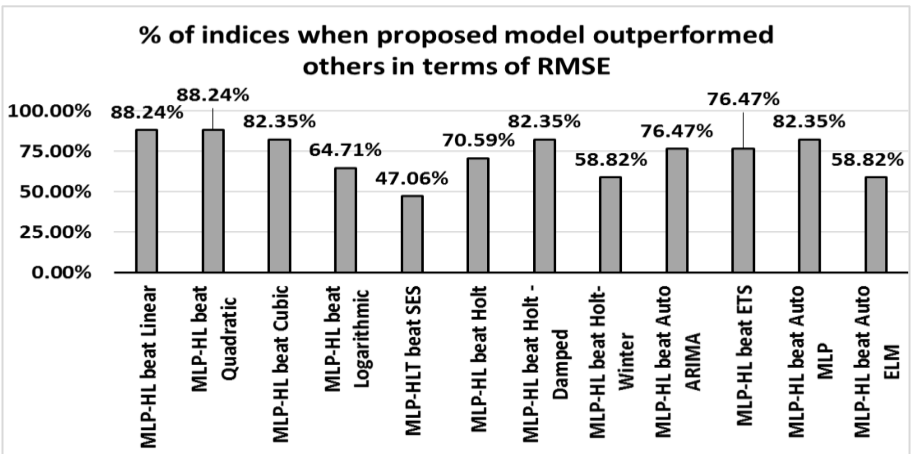

Figure 8 Forecast-RMSE comparison of proposed MLP-HL with twelve different models

In *Figure 9*, *Figure 10* and *Figure 11*, the present work depicts the findings of the performance comparison of the MLP-HL with seven ensemble models. For the majority of the WPIs, the twelve-month forward forecast-accuracies of the proposed MLP-HL were less than the seven ensemble models, i.e., the proposed MLP-HL performed better than them.

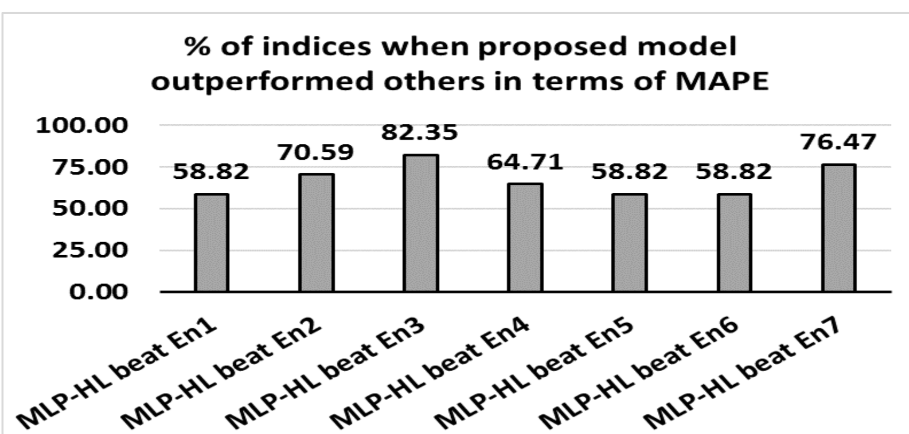

Figure 9 Forecast-MAPE comparison of proposed MLP-HL with seven ensemble models

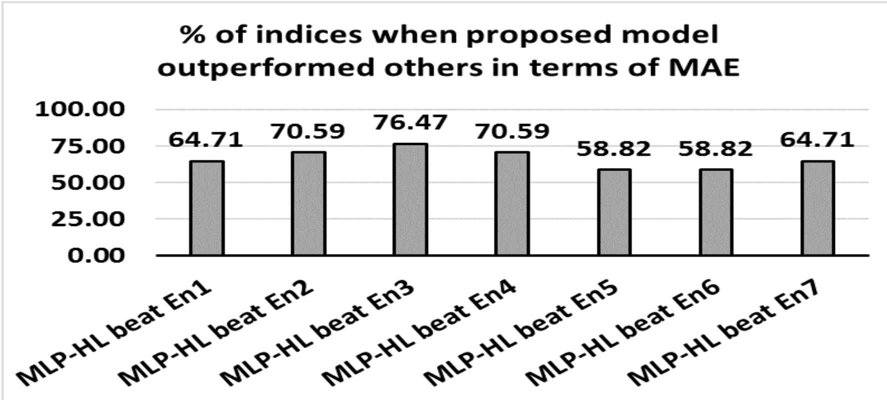

Figure 10 Forecast-MAE comparison of proposed MLP-HL with seven ensemble models

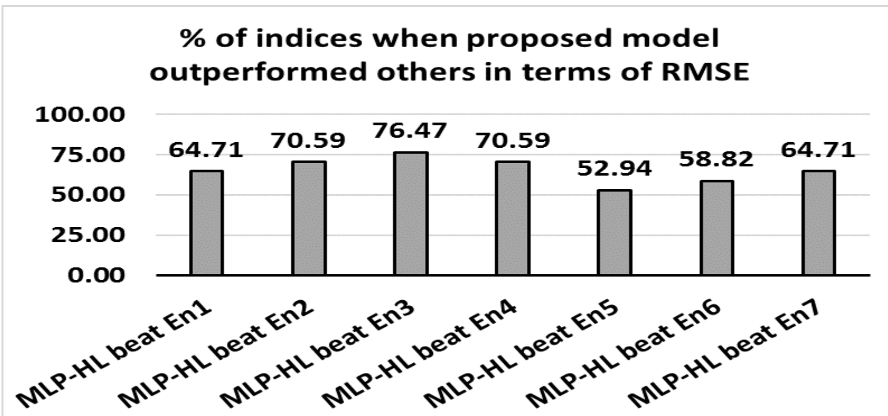

Figure 11 Forecast-RMSE comparison of proposed MLP-HL with seven ensemble models

The present work compared the MLP-HL with the hybrid and ensemble models proposed by the others, and the findings indicate that the MLP-HL's performance is superior (*Table 11*).

Table 11 Comparison of MLP-HL with others

Authors	**Model**	**Application**	**Forecast accuracy metrics**		
			MAPE	**MAE**	**RMSE**
Perone (2020) [64]	Ensemble ARIMA-ETS using CV errors	Forecast hospitalization of COVID 19 mild cases	3.47	113.96	206.49
Perone (2021) [65]	Ensemble ARIMA-ETS using equal weights	Forecast COVID 19 ICU cases	3.47	12.54	21.55
Kumar (2018) [66]	Ensemble ARIMA-ETS	Cloud resource consumption forecast	24.17		
Phuruan and Kasemset (2022) [75]	Hybrid ARIMA-LSTM	Shallot price forecast	13.62	8.51	10.28
Arslan (2022) [76]	Hybrid Prophet-LSTM	Energy consumption forecast		845.16	1106.94
Our work	Ensemble MLP-HL	Forecastin of seventeen non-linear WPIs of MEEQ group	0.52*	0.66*	0.82*

* Best performance obtained for WPI14

5.Discussions

The analysis of forty-eight WPIs using the curve-fit revealed that seventeen indexes (i.e., thirty-five-point four two percent of the total) exhibited non-linear fits. This work identified the type of fit exhibited by an index by inspecting the best fit to the data obtained by applying the statistical rigor-based curve-fit technique. These seventeen indices exhibited heterogeneity also. The work developed the proposed MLP-HL and nineteen different models (eight statistical, four ML, and seven ensemble models) for producing twelve months forward forecasting of these seventeen non-linear indexes. The MLP-HL exhibits a better fit for the majority of the indexes (fifteen out of seventeen indexes), as per the R^2 values. Proposed MLP-HL also offers a high prediction ability for these seventeen indexes according to the AUC scores. The MLP-HL approach exhibits excellent forecast performance for all the seventeen indexes, as observed from the RRMSE values. After computing the forecast-MAPE and forecast-MAE of all the models for twelve months forward forecasting, this paper contrasted the performance of the MLP-HL with others. The comparison demonstrates that the MLP-HL technique outshines others in the majority of instances. The proposed MLP-HL thus proposes a feasible choice and may be applied for a convenient forecast of these seventeen indexes.

5.1Limitations

This work tested the pertinence of the MLP-HL model on seventeen non-linear indexes on which the model exhibited excellent accuracy. This research has not applied the MLP-HL model on those indexes having either linear fit or no fit. Furthermore, the researchers have not tested the MLP-HL model on other univariate non-linear TS datasets for its usability and effectiveness.

A complete list of abbreviations is shown in *Appendix I.*

6.Conclusion and future work

Today, electricity and electrical devices are indispensable, and their role in modern industry is consequential. The manufactured items are the outcome of complicated business processes. These processes heavily depend on electrical devices. Equipment price change impacts the pricing of many other products. Therefore, precise price forecasting for these items aids decision-makers in anticipation of changes, planning, and budgeting ahead of time. Thus, the WPI analysis helps us towards economic understanding, and accurate future value prediction of indexes helps decision-making.

In this research, the authors learned the gaps in the domain of the WPIs from the MEEQ group of the WPI-series of India. While reviewing the recently published works of scholars: constructing forecast models using statistical (regressions, ARIMA, HW, SES, and ETS), ML (MLP, ELM, FB-Prophet, and SVR), and ensemble techniques for the indexes that demonstrate the non-linear fit.

To bridge these gaps, the authors set the following objectives of this work: trend identification of the forty-eight WPIs of the MEEQ group of India’s WPI series using statistical rigor based curve-fit technique; constructing forecast-models for the WPIs containing non-linear fits using regressions, ARIMA, HW, ETS, SES, MLP, ELM, FB-Prophet, SVR, and seven ensemble techniques; proposing a novel MLP-HL based ensemble strategy for the identified non-linear indexes; contrasting the accuracies of the MLP-HL ensemble strategy with the others to assess the MLP-HL model’s forecasting capability.

The authors detailed the novelties of this present work as under:

- Identified the trends of the forty-eight WPIs from the MEEQ group utilizing the curve-fit to specify the WPIs possessing non-linear, linear, and no fits.
- Proposed and developed a novel MLP-HL ensemble strategy capable of delivering twelve months forward forecasts with good accuracy for the WPIs possessing non-linear fits.
- Development of another nineteen-forecasting model (eight statistical, four ML, and seven ensembles) apart from the proposed MLP-HL model for the WPIs possessing non-linear fits.
- Performed proposed MLP-HL and another nineteen developed model's performance comparison.

The curve-fit affirmed that seventeen indexes (out of forty-eight) manifest non-linear fits. This work devised the proposed MLP-HL along with other statistical (eight models), ML (four models), and ensemble (seven models) models for producing forward forecasting of these non-linear indexes. The MLP-HL strategy exhibited excellent forecast performance for all seventeen indexes and outshine others.

The MLP-HL ensemble approach presents a simple, easy-to-implement, efficient, and novel technique for producing more accurate forecasting of the NonLinear_WPIs. In conclusion, the researchers can infer from this work that the offered MLP-HL ensemble model presents a feasible option to forecast them. Validating the suggested MLP-HL model on diverse sets of TS data in the future will reveal insights into its generalization and take us down a new research route.

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Conflicts of interest
The authors have no conflicts of interest to declare.

Author's contributions statements
Dipankar Das: Conceptualization, methodology, data collection, investigation, writing - original draft. **Satyajit Chakrabarti**: Supervision, project administration, review and editing.

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Appendix I

S. No.	Abbriviation	Description
1	AIC	Akaike Information Criteria
2	ANN	Artificial Neural Network
3	AR	Autoregressive
4	ARIMA	Auto-Regressive Integrated Moving Average
5	AUC	Area Under ROC Curve
6	BCG	Bacille Calmette-Guerin
7	CEEMDAN	Complete Ensemble Empirical Mode Decomposition With Adaptive Noise
8	CPI	Consumer Price Index
9	ELM	Extreme Learning Machine
10	ES	Exponential Smoothing

S. No.	Abbriviation	Description
11	ETS	Error, Trend, Seasonal
12	FB-Prophet	Facebook-Prophet
13	HL	Holt's Linear
14	HL_{MAE}	MAE of HL method
15	HL_{MAPE}	MAPE of HL method
16	HW	Holt-Winters
17	LSTM	Long Short Term Memory
18	MAE	Mean Absolute Error
19	MAPE	Mean Absolute Percentage Error
20	MEEQ	Manufacture of Electrical Equipment
21	ML	Machine Learning
22	MLP	Multilayer Perceptron
23	MLP-HL_{MAE}	MAE of MLP-HL method
24	MLP-HL_{MAPE}	MAPE of MLP-Hl method
25	N1	No. of Neurons in the First Hidden Layer
26	N2	No. of Neurons in the Second Hidden Layer
27	NonLinear_WPIs	WPI Series That Show Non Linear Fit
28	Optimized MLP_{MAE}	MAE of Optimized MLP Method
29	Optimized MLP_{MAPE}	MAPE of Optimized MLP Method
30	R^2	Coefficient of Determination
31	RMSE	Root Mean Squared Error
32	RNN	Recurrent Neural Network
33	RRMSE	Relative Root Mean Squared Error
34	SES	Simple Exponential Smoothing
35	SVR	Support Vector Regression
36	TS	Time-series
37	WPI	Wholesale Price Index