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Re-configurable band-stop and all-pass filter using fractional-order topology

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Abstract
The re-configurable filter allows independent tuning of frequency and changes the frequency response of the filter without switching the circuit components. Here, fractional-capacitors and fractional-inductors are used to design a re-configurable fractional order band-stop and all-pass filter to replicate the frequency domain behavior of each other. The proposed filter has been tested in the frequency range of 10 Hz to 10 MHz. The primary aim of this article is to realize, the design parameters (center frequency and bandwidth) of re-configurable filters using fractional order topology, where tunability of filter components is not reliable. To explore the dependence of notches and all-pass characteristics with the variation of fractional exponents of both the proposed filters, the simulation studies have been carried out by varying the exponents (α, β) from 0.1 to 1.0 with a step of 0.1 maintaining one exponent fixed and vice-versa. It is observed experimentally that a re-configurable integer order (IO) band-stop filter can be designed by using fractional value of order (i.e., α=0.5 and β=0.7) of all-pass filter. The stability of the proposed filters has also been investigated by observing the step response of the filter. Finally, the Python software was used to characterize the design parameters of the suggested filters and it was observed that the frequency response of the proposed filters shifts to their IO counterpart at those orders (i.e., α=0.5 and β=0.7). Hence, the flexibility in obtaining the notch frequency in fractional-order all-pass filter using the exponents α and β helps the researchers to realize a band-stop filter with desired specifications.

Keywords
All-pass filter, Band-stop filter, Fractional capacitors, Fractional inductor, Fractional orders, Step response.

1.Introduction
Re-configuration refers to a filter's ability to modify its electrical properties (such as bandwidth, cut-off frequency, phase, etc.) in order to satisfy the dynamic needs of analogue filter design [1–3]. Re-configuration is necessary for complicated electronic systems with one or more stages where a continuous and quick change in the frequency domain is required. Recently, an easy and quick method of switching from one mode to another has been popular in applications of microelectronic systems, signal processing, microwave engineering and etc., [4–6]. Numerous modern technological publications work out the simple task since these re-configurable filters have been overall interest among researchers. These filters provide an effective solution to the demands of emerging research to influence its concerns also in the fields of signal processing and microwave systems [7–10].

Recently, wireless communication systems [11] have used audio signal-processing [12], among other things, and also have reconfigured conventional all-pass and band-stop filters [13–15]. In this context, only the tuning ability [16] of filter components (i.e., resistors, capacitors and inductors) is not enough for adapting to flexible frequency-domain parameters (peak-frequency, bandwidth, etc.). This motivates the authors to realize the re-configuration of all-pass filter to provide notch-responses and that of band-stop filters to give flat magnitude responses [17–19]. However, this important level of reconfiguration cannot be claimed by using any active devices or any integer order (IO) standard passive devices due to their limited degree of design freedom. Therefore, the authors incorporate fractional-order (FO) devices such as fractional capacitor (FC) and fractional inductor (FI) for attaining the required level of re-configurability in active filters. There is a comprehensive design philosophy for filter modelling and its design that starts with filter description and ends with circuit implementation. The introduction of

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FO filters has made it possible to realize any independent order, such as 1.5 or 2.7, whereas the conventional filter design can only handle IO. Radwan et al. (2008, 2009a) introduced design processes for all filters with a fractional-order elements (FOE) when they proposed FO filters [20]. Additionally, general expressions have been derived for peak frequencies, high quality factor, proper phase angle, and the cutoff frequencies.

FOEs are also known as fractance devices, such as FC and FI are employed to realize various analogue filters. Fractance is another name for these FO circuit-based filters with FOE [21]. With the involution of FOEs that offers more freedom in modifying the frequency response, the primary benefits of this type of filter are higher speed of response and operating conditions providing greater flexibility in shaping the frequency response [22]. The requirement of more physical space for hardware components, non-availability of two terminal FOEs for practical implementation and lack of methods for tuning the order of FOEs are some of the drawbacks. However, a number of recent investigations in the specialist literature use analogue filters with FO dynamics [23, 24].

Here, the purpose of this article is to design a re-configurable FO band-stop and all-pass filter to duplicate the frequency domain behavior of each other. The objective of this work is to establish an improved design parameters with re-configurable FO topology, where tunability of filter components (resistors, capacitors and inductors) is not reliable.

In this paper, the realization of all-pass and band-stop filters has been presented by taking two fractional-capacitors (FC1, FC2) and one FC and a single FI respectively. The first half of this section compares the frequency response of the FO band-stop filter and the integer-order (IO) all-pass filter to proceed towards the reconfiguration model. Later, the frequency response of an FO all-pass filter is also contrasted with an IO band-stop filter. Here, a FO all-pass filter is configured using two FCs having orders α and β respectively. On the other hand, a FO band-stop filter is modeled using a FC and a FI bearing orders α and β respectively.

The objective fulfillment of the work done in this study can be summed up as follows:

- A comprehensive analysis of FO band-stop and all-pass filters is discussed in the MATLAB using various combinations of orders (α, β) of FOE.
- The fractional-exponents of the proposed band-stop and all-pass filters are noted which show re-configurable frequency domain characteristics of each other with detailed analysis.
- The percentage of resemblances of both the suggested FO filters with f (α, β) and its design parameters are represented in four dimensional plots using Python software.
- The design parameters of proposed re-configurable filters are evaluated.
- Lastly, the experimental setup of the proposed re-configurable filters with desired fractional-orders (α, β) are implemented using resistance capacitance (RC) symmetric ladder circuit realization of FC.

Furthermore, the document is divided into different sections: The literature review compiled from several articles is in Section 2. Section 3 includes the investigation of the FO all-pass and band-stop filters with their mathematical derivations. Section 4 contains simulation results of the proposed filters. The simulation results that are experimentally validated for the proposed FO filters, comparing their performance of different orders. Section 5 covers the discussion and limitations. The paper is concluded with the probable future work in section 6.

2.Literature review

Depending on the specifications or requirements of a system in a communication channel, re-configurable filters can be applied for modification and gain optimization in a variety of frequency bands [25, 26]. These filters have an essential feature that consists of a single-input single-output topology which depends upon a certain function of the input variables [27]. Several topologies in conventional circuit theory for re-configurable filters have been documented in [28, 29], however due to the simplicity of their construction, higher radio-frequency (RF) bands (GHz range) are given specific attention (i.e., electromagnetically coupled elements) [30]. The switch-less change of transfer response in these RF bands is easily solvable, even with passive solutions [31, 32]. Re-configurable filter research in this area focuses in particular on microwave systems. However, it is not a simple process in the area of low-frequency applications. Therefore, the incorporation of FOE in a filter could justify the reconfiguration of the transfer function in low as well as high-frequency applications as far as frequency-domain analysis is concerned. The authors in [33], discuss a novel electronically re-configurable and tunable fractional-order filters which allows independent frequency tuning and switch-less alteration of transfer response by individual parameter between common band-pass and inverting all-pass response tested in the frequency range of 1 Hz to 100 KHz, whereas, authors in [34], design a FO notch filter for the compensation of the attenuation loss due to order change of the circuit. The authors in [35], established the design of an analogue pseudo-differential fractional frequency filter with an order of (2+α). The suggested filter offers the advantageous features of a completely selective band, but with a simpler circuit topology. This structure operates in mixed-mode featuring both high input and output impedance. However, the future research, will use the fractional-order filters that take account of fractance, which is a unique configuration of FOEs [21].

The present day demands for improvement of design parameters (i.e., bandwidth, selectivity, center frequency, etc.,) in analog filters have increased. Therefore, emerging research focuses on modification of these filters to influence its concerns in the fields of telecommunication [36], signal-processing [37] and microwave [38], etc. So, fractional-order circuits put forward new enhancements such as, establishments of stop-bands and pass-bands with defined higher levels of sharpness than the prototype circuits. In earlier research, most electronic filters were realized using conventional capacitors and inductors. In general, the band-stop filters are primarily used for rejection of unwanted signals and interference in communication systems [39]. The band-stop filter section is designed to reject signals over selected bandwidth and pass all over other ranges. In recent years, two circuit models are most widely used in wireless communication systems having low RF system cost, small size, light weight and high selection features [40]. The all-pass filters are mainly used in phase shifting applications for analog signal processing, where they pass the whole frequency range with a common phase shift, that can be tuned electronically [41]. While re-configuring these filters in fractional domain, the all-pass and band-stop filters become more flexible in shaping the filter responses [42].

Chen et al. [43] presented a novel band-pass to band-stop filter that incorporates surface acoustic wave (SAW) resonators, transmission lines and positive-intrinsic–negative (PIN) diode. As reported in [44] the use of PIN diodes makes it simple to switch from band-pass to band-stop mode by incorporating SAW resonators with transmission systems. The filter illustrates the desirable benefits of high-Q pass-band and stop-band with fractional bandwidths that are significantly wider than 0.4–0.8kt2 for conventional lattice or ladder structure SAW filters. Finally, an electronically switchable band-pass to band-stop SAW filter prototype was designed to validate the proposed concept.

A design of FO integrator was introduced for the operation of the resulting solution in the current mode (CM) as reported in [45]. The integrator's solution is based on the use of RC structures, but unlike other FO designs based on RC structures, the suggested integrator provides electronic control of the exponent of FOE. In addition, unlike typologies based on the approximation of the FO Laplacian operator, the suggested integrator's control does not need numerous precise and exact values of both the control voltages/currents. This integrator also adds the extra function of electronic gain level control.

It was stated in [46] that an inverted impedance multiplier circuit (IIMC) utilizing operational trans-conductance amplifiers (OTA) was used to produce FC and FI for $\alpha > 1$. In [47], a method for developing OTA-based current amplifier-based FO low-pass and high-pass filters with certain common capacitors but no passive resistors were described. The filter developed using this method can be electronically reconfigured, allowing the order and transfer function coefficients that govern the filter's performance to be modified. The usage of universal voltage conveyors (UVC) in the construction of FO low-pass and high-pass filters is once again described by the authors in [48]. Using the continuous fraction expansion (CFE) technique, the transfer functions of these filters were approximated by IO with a transfer function of third order. A generalized FO filter structure with OTAs was suggested, and a resistor-less implementation of these filters with all grounded capacitors was also presented in [49]. These filters implement FO low-pass, high-pass, band-pass, and all-pass filters as well as a FO band-stop filter response.

Alternatively, the authors in [50] proposed a new multi-function fractional-order inverse filter structure that provides optimally infinite input impedance and the fewest active and passive components possible. This is because the proposed structure can realize FO inverse low-pass, high-pass, and band-pass filter using an operational amplifier, two resistors, and two FCs [51]. In terms of reconfiguration, the majority of the research conducted so far is restricted to high frequency applications with different approaches as cited in literature. Additionally, the experiments on fractional-order re-configurable band-stop and all-pass filters by varying the orders of FOE. i.e., FC and FI have been documented in this research work. In this paper, the authors design re-configurable fractional-order band-stop and all-pass filters where the band-stop filter can be transformed to an all-pass filter and vice versa in its frequency domain by merely altering the values of the fractional-exponents to a specific order coefficient.

3.Methods

In this section the proposed filters are designed by using fractional-order elements to investigate FO all-pass and band-stop filter. The objective of this study is to focus on the re-configurable properties of all-pass and band-stop filters when the passive elements (capacitors and inductors) are substituted with their fractional-order counterpart. In both FO filters, the frequency domain analysis is carried-out and the variations in the characteristics and design parameters are observed. As observed from *Figure 1*, the fractional exponents (α, β) of the corresponding filtering system affect the re-configuration property of band-stop and all-pass filters in its frequency domain. Here, the presence of a selective combination of fractional-exponents in the proposed filter transfer function enhances its ability in the frequency domain. *Figure 1* uses the function f(α, β) for a band-stop filter that re-configures it to an all-pass filter and vice versa. The f(α, β) is the function of fractional exponents α, β through which the orders of FOE are varied alternatively keeping other exponents constant. The FO transfer function was approximated in the frequency domain during MATLAB simulation. An 8th stage RC-symmetric ladder circuit of a FC was used to conduct the experimental study. Additionally, experimental work is done to realize the FC for its eighth stage ladder circuit approximation to match the requisite exponent (α, β) values.

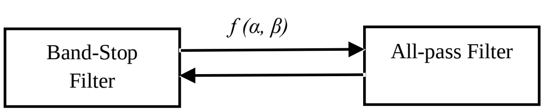


Figure 1 Block-diagram of Re-configurable band-stop and all-pass filter using FO (α, β) topology

3.1Design of fractional-order all-pass filter using two capacitors FC_1 and FC_2

Here, the re-configurable property of all-pass filter is explored when the traditional-capacitors are replaced with its FO counterpart. These two FC such as FC_1 and FC_2 possess their fractional exponents α and β respectively as shown in circuit in *Figure 2.*

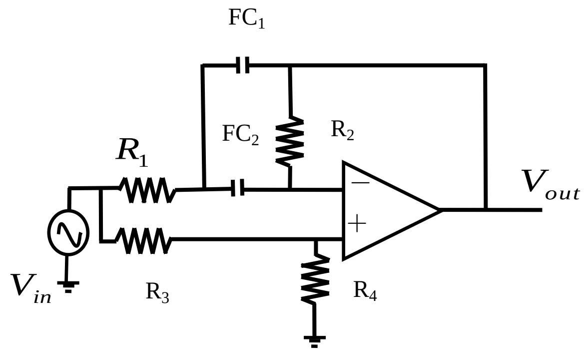


Figure 2 Circuit diagram of integer/fractional-order all-pass filter

Table 1 shows the values of the resistors (kΩ) and capacitors (μF) as the basic components used in simulation of a FO all-pass filter. These components are first taken as IO elements. Further, they are substituted with their FO equivalents after realizing a FC in its RC symmetric approximation model.

Table 1 Component specifications used in simulation of all-pass filter

Components	**R_1**	**R_2**	**R_3**	**R_4**	**FC_1**	**FC_2**
Values	10 kΩ	10 kΩ	10 kΩ	10 kΩ	0.01 μF	0.001 μF

Initially, the fractional capacitors FC_1 and FC_2 are first realized with different fractional exponents, α and β that results in a significant change in its frequency domain behavior. Then, the expression for the transfer function is derived for the filter circuit taking FC_1 and FC_2 of exponents α and β respectively. The transfer function of the FO all-pass filter can be represented in Equation 1.

$$\frac{V_0}{V_i} = \frac{R_4}{R_3+R_4} \cdot \frac{s^{\alpha+\beta} - \frac{2s^{\beta}}{R_2FC_2} + \frac{1}{R_1R_2FC_1FC_2}}{s^{\alpha+\beta} + \frac{2s^{\beta}}{R_2FC_2} + \frac{1}{R_1R_2FC_1FC_2}} \qquad (1)$$

Realizing the FO transfer function in frequency domain, it is stated as Equation 2.

$$\frac{V_0}{V_i} = \frac{R_4}{R_3+R_4} \cdot \frac{(j\omega)^{\alpha+\beta} - \frac{2(j\omega)^{\beta}}{R_2FC_2} + \frac{1}{R_1R_2FC_1FC_2}}{(j\omega)^{\alpha+\beta} + \frac{2(j\omega)^{\beta}}{R_2FC_2} + \frac{1}{R_1R_2FC_1FC_2}} \qquad (2)$$

3.2Design of FO band-stop filter using FC and F

The Band-stop filters are often referred to as band-reject or notch filter, exceed all wavelengths except those within the specified bandwidth. If the stop-band is too small and narrow for a lower range, then the band-stop filter is frequently referred to as a notch filter because, according to its frequency response, it has a considerably deeper gain and a unique feature like an edge curve instead of a broad soft band [18].

Figure 3 illustrates how an FC and FI are employed in an active circuit to model a FO band-stop filter. The implementation of band-stop filter is carried out in integer domain to get its comparison with its fractional counterpart. The conventional capacitors and inductors are being replaced with fractional ones which yields better performance in the form of magnitude and phase response [20].

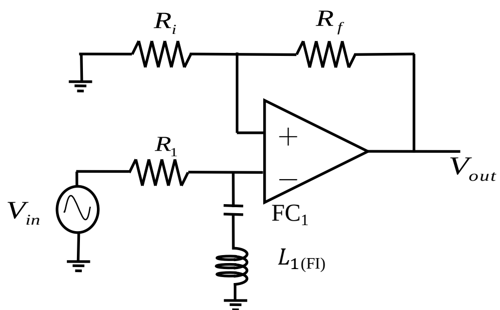


Figure 3 Model diagram of active band-stop filter using FC_1 and FI as FOE

The transfer function representation of FO band-stop filter is expressed as Equation 3.

$$\frac{v_0}{v_i} = \left(1 + \frac{R_f}{R_i}\right) \frac{L_1 s^{\beta} + \frac{1}{FC_1 s^{\alpha}}}{R_1 + L_1 s^{\beta} + \frac{1}{FC_1 s^{\alpha}}} \qquad (3)$$

In frequency domain, it is represented as Equation 4.

$$\frac{v_0}{v_i} = \left(1 + \frac{R_f}{R_i}\right) \frac{L_1 j^{\beta}.\omega^{\beta} + \frac{1}{C_1 j^{\alpha}.\omega^{\alpha}}}{R_1 + L_1 j^{\beta}.\omega^{\beta} + \frac{1}{C_1 j^{\alpha}.\omega^{\alpha}}} \qquad (4)$$

The Equation 4 is further simplified and separated for magnitude and phase expressions. Equation 5 is derived for magnitude response and Equation 6 represents the phase response of a FO band-stop filter. The magnitude of the FO band-stop filter is expressed as Equation 5.

$$|mag| = \left| \frac{\{1 + FC_1 L_1 \omega^{\alpha+\beta} \cos(\alpha+\beta)\} + j\left(FC_1 L_1 \omega^{\alpha+\beta} \sin(\alpha+\beta)\right)}{\{R_1 FC_1 \omega^{\alpha} \cos\alpha + FC_1 L_1 \omega^{\alpha+\beta} \cos(\alpha+\beta)\} + j\{R_1 FC_1 \omega^{\alpha} \sin\alpha + FC_1 L_1 \omega^{\alpha+\beta} \sin(\alpha+\beta)\}} \right| \qquad (5)$$

Again, from the equation 4, the phase of the filter is derived as Equation 6.

$$phase\ (\phi) = \tan^{-1}\left(\frac{R_1 FC_1 \omega^{\alpha} \sin\alpha + FC_1 L_1 \omega^{\alpha+\beta} \sin(\alpha+\beta)}{R_1 FC_1 \omega^{\alpha} \cos\alpha + FC_1 L_1 \omega^{\alpha+\beta} \cos(\alpha+\beta)}\right) \qquad (6)$$

Table 2 shows the values of the resistors (kΩ) inductor (mH) and capacitor (μF) as the basic components used in simulation of a FO band-stop filter. These components are first taken as integer elements. Furthermore, they are replaced with their FO counterparts. The component values used in simulation of band-stop filter are taken for the 8th stage in the RC symmetric approximation model of a FC.

Table 2 Component specifications used in simulation of band-stop filter

Components	R_1	Ri	R_f	L_1	FC_1
Values	100 kΩ	10 kΩ	20 Ω	0.01 mH	0.001 μF

4.Results

The gain and phase along with frequency dependent parameters like bandwidth, centre frequency was simulated in MATLAB. These results are plotted in 4-D to prove the reconfiguration property of these two proposed filters. The simulation is carried out by taking the exponent values, keeping one of them constant and the other variables in the range of 0.1 to 1.0.

4.1Analysis of FO all-pass filter using two capacitors FC_1 and FC_2

In MATLAB, the magnitude and the phase responses were produced by varying fractional-orders of each component used in the filter circuit, alternatively maintaining another order of FOE as unchanged.

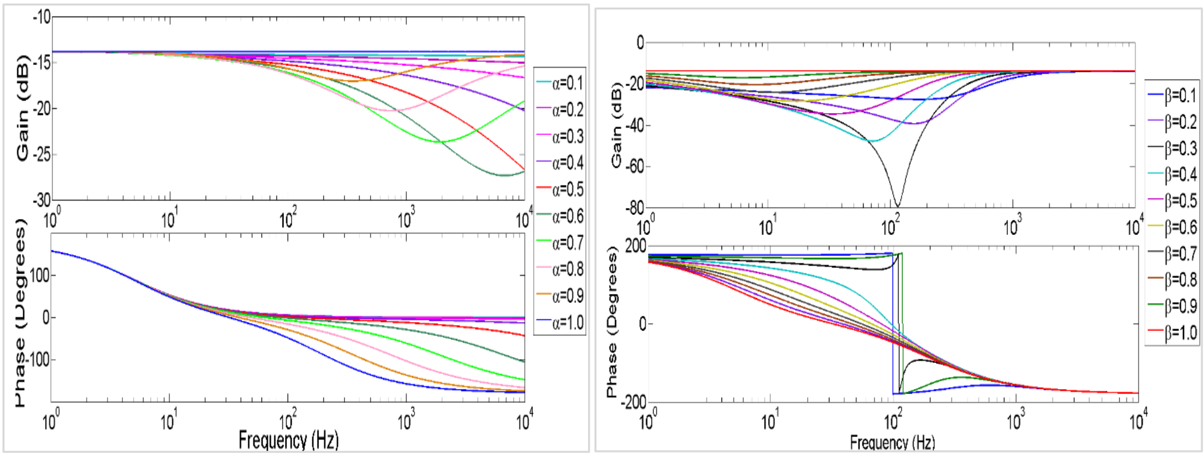
Case -I: α variable and β constant

In this case, the exponent value, α of the FC is increased from 0.1 to 1 maintaining other exponent β of the FI fixed (i.e. β=1). The gain (dB) and phase (degrees) of the filters are simulated over the range of exponents 0.1≤ α ≤1.0 and plotted in *Figure 4(a)*

Case -II: β variable and α constant

For this case the exponent value, β of FI is changed from 0.1 to 1 keeping other exponent α of FC constant (i.e., α=1) The gain (dB) and phase (degrees) are simulated over the range of exponents 0.1≤β≤1.0 and plotted in *Figure 4(b).*

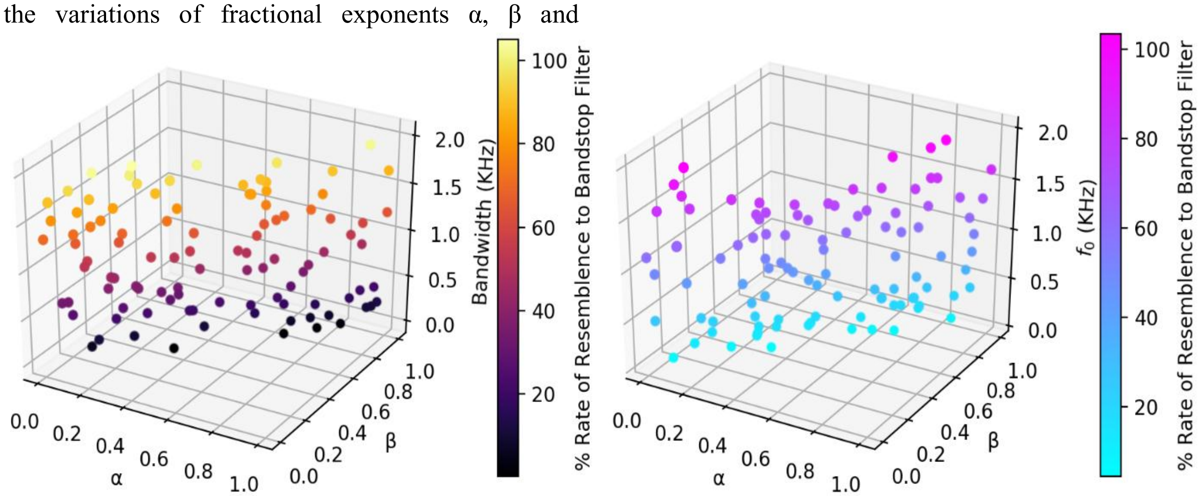
Here, it is observed that in *Figure 4*(a) at α=0.5, β=1 and in *Figure 4(b)* at β=0.7, α=1 the frequency-domain behavior of FO all-pass filter re-configured, that provides band-stop filter response. Using Python software, the magnitude of the proposed FO filter is obtained from Equation 2. Then the centre frequency and bandwidth are numerically evaluated by equating the magnitude to -3dB and applying conditions, $\frac{d}{d\omega}|G(j.\omega)|_{\omega=\omega_c} = 0$ in Equation 2.



(a) (b)

Figure 4 Frequency response of FO all-pass filter (a) Keeping α variable and β constant (b) Keeping β variable and α constant

The studies shown in *Figure 5* show the reconfiguration behavior of frequency parameters (center-frequency and bandwidth) of the FO all-pass filter resembling a band-stop filter. *Figure 5(a)* plots the variations of fractional exponents α, β and bandwidth w.r.t rate of resemblance to band-stop filter and *Figure 5(b)* shows the variations of fractional exponents α, β and centre-frequency w.r.t rate of resemblance to band-stop filter.



(a) (b)

Figure 5 4-D plot showing frequency response of FO all-pass filter with IO band-stop filter by varying orders α and β. (a) Bandwidth vs. % rate of resemblance (b) Centre frequency (f_0) vs. % rate of resemblance

Here, the simulated MATLAB results in *Figure 4* have been validated by the numerical analysis shown in *Figure 5* through Python. i.e., at α =0.5 and β= 0.7. It has been remarked that the FO all-pass filter transforms into an IO band-stop filter at these fractional exponents.

4.2 Analysis of fractional-order band stop filter using FC and FI

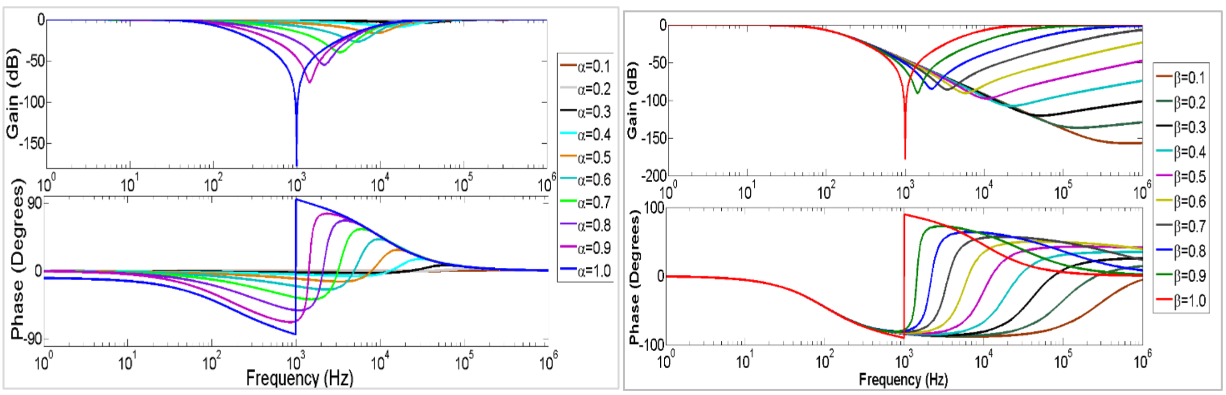
The frequency domain analysis is performed from the above magnitude and phase expressions. The FO band-stop filter is simulated in MATLAB by varying the orders α and β of the FOEs. The *Figure 6(a)* represents the magnitude/gain (dB) and phase response of active band-stop filter with β=1 and α varying from 0.1 to 1.0 and vice-versa.

At α =0.1, the frequency response of FO band-stop filter re-configures to IO all-pass filter. From *Figure 6(a)* and *6(b),* the FO band-stop filter flattens its response with a decrease in β i.e., β=0.3 to 0.1, it does not reconfigure the IO all-pass filter, rather the response matches with the FO all-pass filter when α= 0.7 to 0.9 and β= 0.6 to 0.9. respectively. *Figure 6(b)* represents the magnitude and phase response of FO band-stop filter with change in order β from 0.1 to 1.0 keeping α=1. From Equation 4, the magnitude of the suggested FO filter is obtained by using Python software. Then the centre frequency and bandwidth are numerically evaluated by equating the magnitude to -3dB and applying condition, $\frac{d}{d\omega}|G(j.\omega)|_{\omega=\omega_c} = 0$ in Equation 4.

The frequency parameters (i.e., center-frequency, bandwidth) of a FO band-stop filter that resembles an all-pass filter were evaluated in order to investigate the filter's re-configuration behaviour in *Figure 7*.

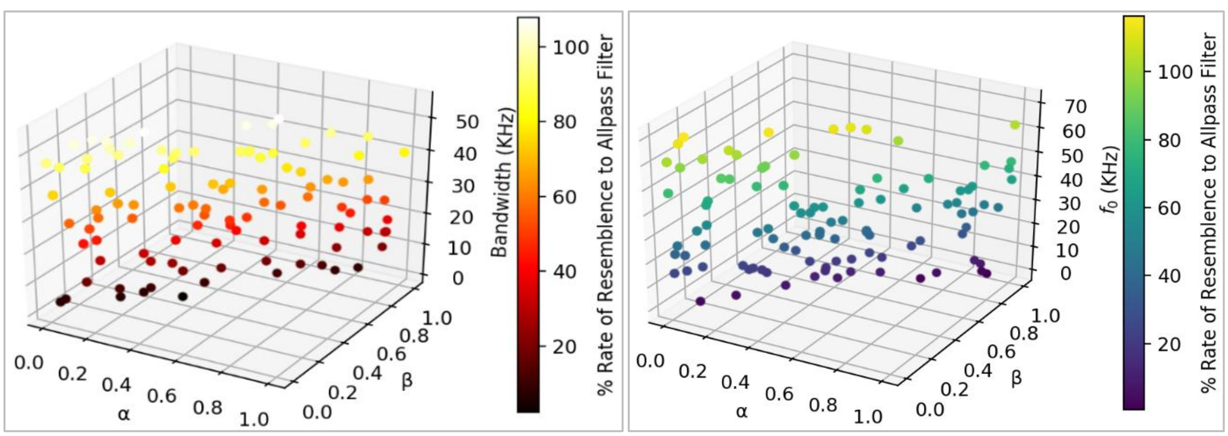
(a) Variations of fractional exponents α, β and bandwidth w.r.t rate of resemblance to an all-pass filter.

(b) Variations of fractional exponents α, β and center-frequency w.r.t rate of resemblance to an all-pass filter.



(a) (b)

Figure 6 Magnitude and Phase response of active band-stop filter (a) Keeping β constant and α variable (b) Keeping α constant and β variable



(a) (b)

Figure 7 4-D plot showing frequency response of FO band-stop filter with IO all-pass filter by varying α and β. **(a)** Bandwidth vs. % rate of resemblance **(b)** Centre frequency (f_0) vs. % rate of resemblance

As a result of the study conducted in this subsection, when the orders α, β are decreased from 1.0 to 0.1, the band stop filter's parameters in its magnitude response can be reconfigured to become those of an all-pass filter. In MATLAB, it is evident that the FO band stop filter attempts to duplicate the response of the all-pass filter where the results in *Figure 6* is observed at α=0.1 to α=0.3. This result is supported by its magnitude response, which is given in Equation 4, when compared to the results of Python simulation shown in *Figure 7*.

4.3Experimental validation

Now, a FC approximated by an RC symmetric ladder is employed in place of a conventional capacitor to design FO filter circuits as specified. Here, the desired order of FC is developed using an RC symmetric ladder model with an 8th stage approximation for implementation in the filter circuit. The authors modified Foster-I representation is used to find the values of the resistors and capacitor for the required order of FC [37]. Here, the structural model consists of one resistor connected in series with N+2 parallel RC sections scaled as 0, 1, 2,...,N+2. An approximated RC symmetric ladder circuit representation for a FC was designed in breadboard as shown in the circuit below.

The approximated FC circuit model is depicted in *Figure 8*. In this case, the cascading of RC components results in a ladder network that provides an effective reactance with an approximate frequency domain characteristic of FC. The order and capacitance values of the FC are determined by the values of the capacitors and resistors, which are also documented in a table.

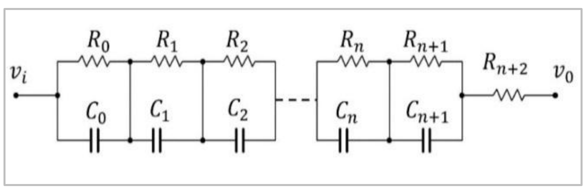


Figure 8 RC symmetric ladder model of a FC

Table 3 provides a list of the probable R and C values that could be used in the RC symmetric ladder circuit approximation model for a FC of desired orders. The values of the resistors (R) and capacitors (C) are tabulated in an 8th stage RC symmetric ladder circuit for a FC while taking account for the various exponent values (i.e., α=0.5 and β=0.7) at which the conversion from a fractional-order band-stop filter to a fractional-order all-pass filter exists.

Table 3 Component specifications in RC symmetric model for FC of desired orders

Components	**β=0.7**	**α=0.5**
R_0	2 MΩ	15 kΩ
C_0	50 μF	64 μF
R_1	208 kΩ	38 kΩ
C_1	51 μF	38 μF
R_2	43 kΩ	1.5 kΩ
C_2	26 μF	15 μF
R_3	9 kΩ	0.5 kΩ

Now, the approximate fractional capacitors of α =0.5, β=0.7 is designed in breadboard. Then this sub circuit replaces the conventional capacitors in the all-pass filter in two cases as seen in the simulation analysis. This experiment yields the frequency response which is compared to the simulated results. *Figure 9* displays a photo of the full experimental setup. In this context, a breadboard was used to design the RC symmetric approximation FC as depicted in *Figure 8* employing the combination of R and C values. The two fractional orders (FO) of FC (α=0.5 and α=1.0) are taken into consideration and experimented in the arrangement as shown in the laboratory. The setup is arranged with a function generator, oscilloscope, RC components and a breadboard. A proper connection is established to input a signal from the function generator, and the output voltage waveform is displayed and recorded on the oscilloscope screen.

Figure 9 Experimental setup for realization of FO all-pass filter using RC-symmetric ladder model

The experiment was carried-out by taking two approximated FCs of orders α=0.5 and β=0.7 to realise a FO all-pass filter. The magnitude response obtained in *Figure 10* depicts the similarity of the experimental output with the simulated results. The frequency parameters like centre frequency, lower and higher cut-off frequency, bandwidth and quality factor are tabulated for two FCs at α=0.5, β=1.0 and α=1.0, β=0.7 respectively. At these exponent values, the simulated results are compared with experimental outputs. The results are thereafter validated experimentally at α=0.5 and β=0.7 for a FO all-pass filter which resembles a band-stop filter's response.

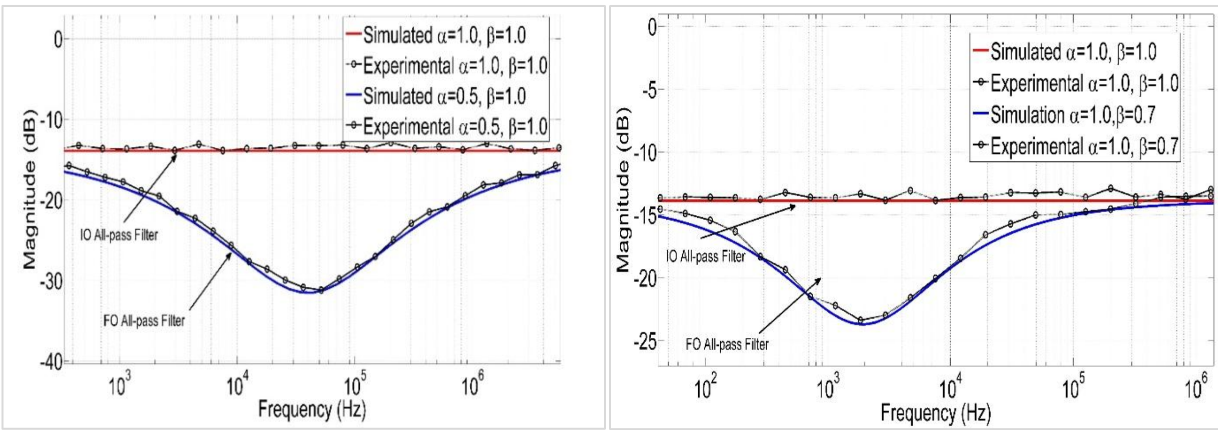
Here, the experimental analysis conducted using RC symmetric approximated FC provides similar results. Observing the data tabulated in *Table 4* clearly shows a marginal difference is observed in *Figure 10*, which is mainly due to use of an approximated FC, not a FC of ideal characteristics. Rather, an 8^{th} order approximation is initiated experimentally to generalize this FC. If an RC symmetric model could be designed of an infinite ladder, it could exactly duplicate an ideal FC.

4.4Stability analysis

The time domain analysis of the suggested filters has been conducted by applying a step input to the filters [8, 12]. *Figure 11*, shows the stability analysis of the FO all-pass filter and was studied for multiple orders of FOEs. Here, in the first case β is fixed, i.e., β=1 and α vary from 0.3 to 1.0. Secondly, the fractional-order α is kept constant and β varies from 0.3 to 1.0.

Again, in *Figure 12*, the stability analysis of the FO band-stop filter is being explored. In all the cases, the fractional exponents have always been accepted as greater than 0.3 since other exponent values are below 0.3 give inaccurate results.

A complete list of abbreviations is shown in *Appendix I.*

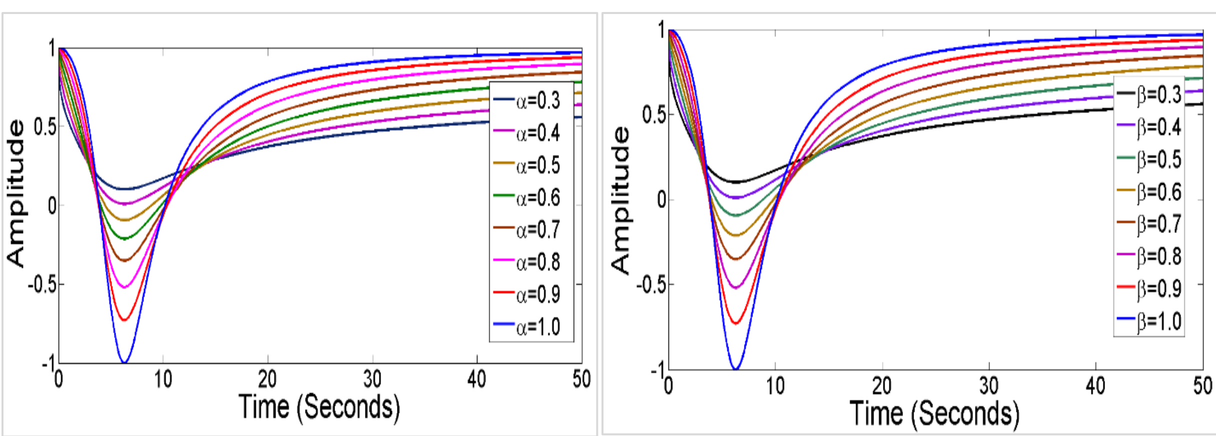


(a) (b)

Figure 10 Comparison of results for a FO all-pass filter at **(a)** α=0.5 and β=1.0, **(b)** α=1.0 and β=0.7

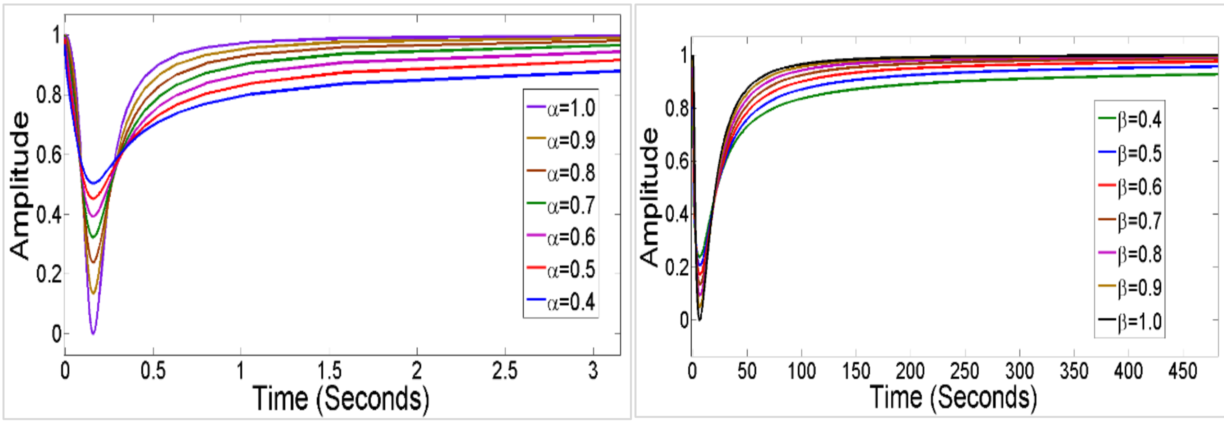
Table 4 Comparison of results for a FO all-pass filter

Frequency parameters	(α=0.5, β=1.0) Simulated	(α=0.5, β=1.0) Experimental	(α=1.0, β=0.7) Simulated	(α=1.0, β=0.7) Experimental
Centre frequency	44.31 KHz	59.4 KHz	1973 Hz	2217 Hz
Lower cut-off frequency	13.85 KHz	17.03 KHz	591 Hz	682 Hz
Higher cut-off frequency	112.3 KHz	136.7 KHz	6010 Hz	7488 Hz.
Bandwidth	98.5 Hz	119.67 Hz	5491 Hz	6808 Hz
Quality factor	0.44	0.49	0.35	0.32



(a) (b)

Figure 11 Step response of FO all-pass filter (a) at α=0.3 to 1.0 and (b) β=0.3 to 1.0



(a) (b)

Figure 12 Step response of FO band-stop filter (a) at α=0.3 to 1.0 and (b) β=0.3 to 1.0

5.Discussion

Here, the addition of FO elements enables the band-stop filter to be re-configured into an all-pass filter, and vice-versa. Conventionally, IO band-stop filters have a notch response and FO band-stop filters have a flatter frequency response because the selectivity is reduced as the bandwidth is increased by varying the exponents of the FOE from 1.0 to 0.1. Finally, the band-stop filter transforms into an all-pass filter with a flat frequency response upon additional modification of these exponents to α=0.1 when β=1.0, and β=0.3 when α=1.0. In addition to this, when the orders of FOE are reduced from 1.0 to 0.1, a spike in the all-pass filter response appears at α=0.5, β=1.0 and α=1.0, β=0.7, which resembles band-stop filter response.

In this research, the reconfiguration of the band-stop filters to an all-pass filter as well as the reconfiguration of the all-pass filter into a band-stop filter has been carried out, without modifying its transfer function. Rather, in this case, due to the variation of the fractional-exponents of the filter components (i.e., fractional-capacitors and FI) the frequency domain behavior of band-stop filter and all-pass filter interchange with each other. As it is difficult to achieve this re-configurable property in all-pass and band-stop filters using conventional resistance inductance capacitance (RLC) components in any electric circuit, therefore, it is advisable to reconfigure these filters using fractional-order topology. The comparison of the FO all-pass filter and the FO band-stop filter has been studied in terms of their percentage of resemblance with each other. Here, the percentage of resemblance at every fractional-order combination has been shown in 4D responses in *Figure 5* and *Figure 6* using python software. This comparative analysis shows the re-configurable property of the proposed filter using fractional-order topology. However, this important level of re-configurability has not been reported yet. RC-Symmetric FCs were used to carry out the experimental validation of the proposed re-configurable filters. Where, it has been found that the experimental data converges more or less with the simulated results. This slight deviation of the experimental data is due to the non-idealities of FO components (i.e., approximation of FCs) and experimental errors, etc.

5.1Limitations

Since the process of reconfiguration is difficult to accomplish the requisite design parameters of the filter, the commercialization and production of FC_S and inductors with desired parameters haven't really emerged yet. A RC symmetric model for approximating FC, which behaves as a constant phase element in a limited frequency range, is thus employed in this work. If the FC had been approximated to an infinite stage in the RC symmetric ladder model, it would have offered better results. However, the FC or FI so far practically used in literature are not tunable in nature. Whereas, in physical terms the re-configurable filters using fractional topology can only be used if practically tunable FO devices are implemented. This is one of the major limitations of this work.

Moreover, the majority of this work is confined to numerical analysis and simulation studies.

6.Conclusion and future work

In this study, the conventional all-pass and band-stop filters were simulated in MATLAB R2011a where, the inductors and the capacitors were replaced by its FO counterpart. In these proposed filters, one of the order of FOE is kept constant (i.e., α=1), while the other order, β varies from 0.1 to 1.0 with a step of 0.1 and vice-versa. It was observed and validated experimentally that a re-configurable IO band-stop filter can be designed by using fractional exponents (i.e., α=0.5, β=1.0 and α=1.0, β=0.7) of all-pass filter. Whereas, in FO band-stop filter, at α=0.1 to α=0.3 keeping β=1, it resembles the response of the IO all-pass filter.

Meanwhile, the magnitude response of band-stop filter flattens with the decrease in β value within the range of 0.1 to 0.3 keeping α=1.0, and matches with the FO all-pass filter of orders, α= 0.7 to 0.9 at β=1.0 and β= 0.6 to 0.9 at α=1.0. These observations were remarked in MATLAB simulations and also justified by 4D plot showing frequency response obtained in Python. The design of a re-configurable band-stop and all-pass filter could be implemented and its future uses are in the fields of wireless communication, radar technology etc. Presently tunable fractional elements are found in many applications of re-configurable filters, whereas just varying the orders of FOE, which would be definitely preferable than replacing the entire filter in re-configuration topology. The use of these tunable fractional elements using field programmable gate array (FPGA) technology would also enhance the filter stability and reduce the effect of environmental noise and interference.

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Conflicts of interest
The authors have no conflicts of interest to declare.

Author’s contribution statement
Mr. Kumar Biswal: Conceptualization, Investigation, Data curation, Writing – original draft, Writing – review and editing. **Sumit Swain:** Carried out the comparative analysis and set up the experiments for the research work, Writing – review and editing. **Madhab Chandra Tripathy and Sanjeeb Kumar Kar**: Supervision, Investigation on challenges, Writing – review and editing.

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Appendix I

S. No.	Abbreviation	Description
1	CFE	Continuous Fraction Expansion
2	CM	Current Mode
3	FC	Fractional Capacitor
4	FI	Fractional Inductor
5	FO	Fractional Order
6	FOE	Fractional Order Element
7	FPGA	Field Programmable Gate Array
8	IIMC	Inverted Impedance Multiplier Circuit
9	IO	Integer Order
10	OTA	Operational Transconductance Amplifier
11	PIN	Positive-Intrinsic–Negative
12	RC	Resistance Capacitance
13	RF	Radio Frequency
14	RLC	Resistance Inductance Capacitance
15	SAW	Surface Acoustic Wave
16	UVC	Universal Voltage Conveyors