

# A study of transient effect of constant indirect flow velocity through multiple upper-vents in un-stratified rectangular ventilated building using theoretical approach

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## Abstract

*The paper is an extension of authors own work in which, studied of transient effect of stack-driven airflow in cross-ventilated building with three opening in the presence of opposing flow in one of the upper-vent. An analysis has been carried out to study the transient effect of constant indirect flow velocity in rectangular building with multiple upper-vents induced by stack-driven effect. Moreover, equations of momentum and energy are non dimensionalised using some dimensionless quantities and solved theoretically by means of separation of variable method. The asymptotic behavior of parameters involved in the study predicts the result for Velocity, temperature distributions together with volumetric and mass- transfer. The results of the study are presented graphically and discussed for varying values of physical parameters involved such as, effective thermal coefficient( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ). In addition, comparison with previously published work by was performed. In which, the study concluded that, the results for present work is more effective and efficient than the previous work in term of ventilation process. Finally, from the course of investigation, it was observed air temperature and velocity increase with the increase in both parameters( $\theta_0$ ), ( $Pr$ ) and ( $Gr$ ) respectively.*

## Keywords

*Transient effect, Indirect flow velocity, Multiple upper vents, Ventilated building.*

## 1.Introduction

Studied associated to natural convection flow of incompressible and compressible fluids has received considerable interest due to the enormous applications in various fields of industry, architectural design, science and technology. Several studies have been reported on natural convection flow under different physical situations. A study of natural ventilations in building plays an important role in architectural design especially, in building envelopes. Airflow process can either be achieved by natural means (natural ventilation) or by some external means such as, fan, air conditioners etc. (mechanical ventilation) or by combined natural and mechanical ventilations (hybrid ventilation).

Many investigation and experiments have been carried out by previous researchers in ventilation phenomena. Some of the previous interventions in the area are; [1] investigated airflow process in single-sided building. Performed an experiment on scale effect in room air-flow and later [2, 3] investigated air movement on naturally-ventilated building. Investigated air flow across wall vents caused by thermal source in building based on the study given by [4, 3].

Studied the effect of buoyancy forces on airflow across the two openings in building [5]. Studied the effect of indirect flow with constant indirect velocity in rectangular ventilated building with three- vents [6]. Studied a building with bi-directional flow openings [7]. Developed a computational fluid dynamical model in rooms with indoor air pollutant

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[8]. Investigated a room airflow distribution system using computational fluid dynamic (CFD) [9]. Described the approach with advantages and disadvantages of various methods for modeling air flow in the building and also studies buoyancy-driven natural ventilation of buildings-impact of computational domain [10]. Studied displacement ventilation when the interior of the building was stratified[11]. Studied the mixed ventilation when the interior of the building has uniform temperature [12]. Considered airflow process combining the ideas of mixed and displacement ventilations in building by [13, 14]. Studied effect of stack- driven airflow in cross- ventilated building with an opposing flow in one of the upper- vent [15]. Developed a mathematical modeling of wind forces [16].

Developed a linear thermal models of mixed- mode building [17]. Examined airflow process induced by buoyancy- driven forces on the floor of an enclosure in the presence of wind [18]. Estimated exchange of air by natural means potential and considered thermal comfort issues [19]. Performed an experiment examining the heat-transfer and airflow in interactive building façade [20]. Studied natural convection flow for heat and mass- transfer in a single- sided ventilated building [21]. Performed CFD simulation to reproduce the decay of  $CO_2$  concentration in a large semi-enclosed stadium [22]. Developed natural Ventilation potential model of airflow in china [23]. performed an experiment in heated, sealed room of a test house [24].

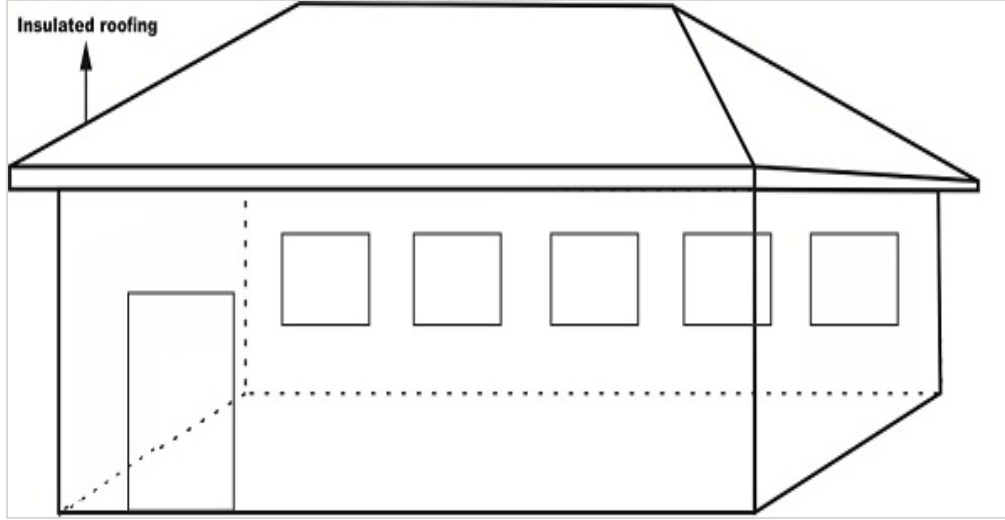
Performed an experiment on a vertical temperature distribution by combining natural and mechanical ventilation in an atrium building[25]. Performed an experiment and investigated effect of buoyancy-driven forces in single- sided ventilated building with large openings by means of CFD methods[26]. Performed an experiment in full size ventilated box [27]. Studied a building with two openings at different vertical height [28]. Studied airflow caused by wind and buoyancy forces [29]. Studied the effect of wind- driven flow only in building interacted with wind [30]. Studied the effect of wind- driven flow in building envelope with small openings [31]. Developed a model of airflow induced by stack-driven effect in multi-compartment buildings. Calculation of heat- transfer and energy balance of a

double skin façade (DSF) was studied by [32, 33]. Effect of two buoyancy forces on natural ventilation in an enclosure was investigated by [34]. Studied a natural convection flow in a single sided building with partition-I [35]. Studied a natural convection flow in a single sided building with partition-II [36]. Studied fluid mechanics of natural ventilation[37]. Performed an experiment in a full- scale naturally ventilated building [38]. Studied transient investigation of airflow through two upper vertical- vents in the absence of constant indirect flow velocity in rectangular building [39]. Investigated the potential use of natural ventilation strategies in high-rise building in summer [40]. Developed a nodal model to estimate temperature stratification in rooms with displacement ventilation [41]. Discussed numerical simulation of the wind velocity in individual building using multi- zone modelling [42]. Studied the impact of window parameters on the building [43].

The objectives of this study is presents the model and discuss the effect of parameters and other operating conditions involved in the study to the effect of constant indirect flow velocity in rectangular building with multiple- upper vents induced by stack- driven effect. The governing equations describing the flow are written in dimensionless form and solved theoretically by means of separation of variable method. In this paper the velocity, temperature distributions together with mass- transfer and volumetric airflow are obtained and discussed for some selected values of parameters, such as Effective thermal coefficient( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ). Therefore, the model is only limited with vertical openings on the same height.

## 2.Domain description

The paper considers a natural convection flow in unstratified rectangular building with multiple- upper vents induced by stack- driven airflow. The building envelope has air as the connecting fluid and separated from one another by vertical- vents of height( $y^*$ ) and constant width of the vents( $x_w$ ) which is shown in *Figure 1*. The density, velocity, temperature and pressure of air maintained at  $\rho_a, U^*, T^*$  and  $P$ .

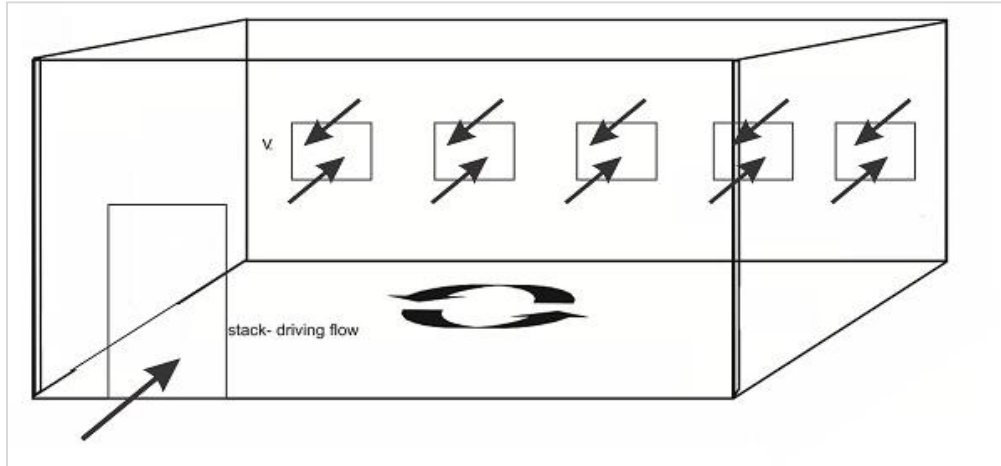


**Figure 1** Diagram of the building envelope

### 3. Model formulation

The preliminary assumptions worked in the study are, flow is assumed to be depend on the height of the vents, steady flow with no internal source so as the density of air will be nearly constant like

incompressible fluid ( $\rho_a \approx \text{constant}$ ) and the pressure be a component along the width of the vents in the building. The airflow induced by Stack- driven effect is shown in *Figure 2*.



**Figure 2** Schematic diagram of airflow in building induced by stack- driven effect

Under the usual assumption of reduced gravity, the governing equations in dimensional form of the momentum and energy equations are,

$$\frac{\partial u}{\partial t} + v_0 \frac{\partial u}{\partial y} = g\beta\Delta T + \nu \frac{\partial^2 u}{\partial y^2}. \quad (1)$$

$$\frac{\partial T}{\partial t} + v_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}. \quad (2)$$

with the initial and boundary conditions that satisfy the problem,

$$v_0 = \text{const.}, u = 0, u(0, t) = 0, T = -\theta_0, T(0, t) = 0 \text{ at } y = 0 \text{ and } u = 0, u(1, t) = 0, T = 1 - \theta_0, T(1, t) = 0 \text{ at } y = 1. \quad (3)$$

Scaling  $y$  with  $YL$ , velocity  $u$  with  $\frac{U^* g\beta\Delta\theta L^2}{\alpha}$ ,  $t = \frac{t^* L^2}{\alpha}$ , and introducing  $T$  with  $T^* \Delta T + T_a$ .

The governing equations in (1), (2) and (3) are transformed into dimensionless form as,

$$\frac{\partial U^*}{\partial t^*} - C \frac{\partial U^*}{\partial Y} = Pr \frac{\partial^2 U^*}{\partial Y^2} + PrGrT^*(Y, t^*) \quad (4)$$

$$\frac{\partial T^*}{\partial t^*} - C \frac{\partial T^*}{\partial Y} = \frac{\partial^2 T^*}{\partial Y^2} \quad (5)$$

with the following dimensionless boundary conditions as,

$$U^* = 0, U_u^*(0, t^*) = 0, T^* = -\theta_0, T_u^*(0, t^*) = 0 \text{ at } Y = 0 \\ \text{and } U^* = 0, U_u^*(1, t^*) = 0, \frac{\partial U^*(1, t_{max})}{\partial Y} = U_0, T^* = 1 - \theta_0, T_u^*(1, t^*) = 0, T_u^*(1, t_{max}) = \text{shint}^* \text{ at } Y = 1. \quad (6)$$

where,  $C = -v_0 Pr$ .

The steady state equation for (5) is,

$$\frac{d^2 T^*}{dY^2} + C \frac{\partial T^*}{\partial Y} = 0 \quad (7)$$

solving equation (7) and applying boundary condition (6), we obtain steady state solution for temperature distributions as,

$$T^*(Y) = -\theta_0 + \frac{e^C}{1-e^C} (e^{-CY} - 1) \quad (8)$$

The unsteady state equation for (5) is maintained as,

$$\frac{\partial T^*}{\partial t^*} - C \frac{\partial T^*}{\partial Y} = \frac{\partial^2 T^*}{\partial Y^2}$$

steady and unsteady part of solution is,

$$T^*(Y, t^*) = T^*(Y) + T_u^*(Y, t^*) \quad (9)$$

The separation is valid for unsteady part solution as,

$$\frac{\partial T_u^*}{\partial t^*} - C \frac{\partial T_u^*}{\partial Y} = \frac{\partial^2 T_u^*}{\partial Y^2} \quad (10)$$

solving equation (10) and applying boundary condition (6), we obtain the unsteady state solution for temperature distributions as,

$$T_u^*(Y, t^*) = e^{-(P_1^2 t^* + \frac{C}{2})} (C_1 \cosh dY + C_2 \sinh dY) \quad (11)$$

which can be further simplify and obtains,

$$T_u^*(Y, t^*) = \frac{\text{shint}^*}{\cosh d} e^{(P_1^2 (t_{max} - t^*) + \frac{C}{2})} \cosh dY \quad (12)$$

$$\text{where, } C_1 = 0, C_2 = \frac{\text{shint}^*}{\cosh d} e^{(P_1^2 t_{max} + \frac{C}{2})} \text{ for } d = \frac{\sqrt{C^2 - 4P_1^2}}{2}, \\ 0 < P_1 \leq \frac{C}{2} \text{ at } t^* \geq 0.$$

Therefore, the temperature distributions across the vents is,

$$T^*(Y, t^*) = -\theta_0 + \frac{e^C}{1-e^C} (e^{-CY} - 1) + \frac{\text{shint}^*}{\cosh d} e^{(P_1^2 t_{max} - t^*)} \cosh dY \quad (13)$$

The unsteady state equation for (4) is maintained as,

$$\frac{\partial U^*}{\partial t^*} - C \frac{\partial U^*}{\partial Y} = Pr \frac{\partial^2 U^*}{\partial Y^2} + PrGr T^*(Y, t^*) \\ \frac{\partial U^*}{\partial t^*} - C \frac{\partial U^*}{\partial Y} = Pr \frac{\partial^2 U^*}{\partial Y^2} + PrGr \left( -\theta_0 + \frac{e^C}{1-e^C} (e^{-CY} - 1) + \frac{\text{shint}^*}{\cosh d} e^{(P_1^2 t_{max} - t^*)} \cosh dY \right) \quad (14)$$

The steady state equation of (14) is,

$$Pr \frac{\partial^2 U^*}{\partial Y^2} + C \frac{\partial U^*}{\partial Y} = PrGr \left( -\theta_0 + \frac{e^C}{1-e^C} (e^{-CY} - 1) \right) \quad (15)$$

solving equation (15) and applying boundary condition (6), we obtain the steady state solution for velocity distributions as,

$$U^*(Y) = C_3 + C_4 e^{v_0 Y} + \frac{Gr}{C^2(1-e^{-C})} \left[ (\theta_0(1-e^{-C}) - 1)CY - e^{-CY} \left( 1 + \frac{Pr}{1-Pr} \right) + Pr(1-\theta_0(1-e^{-C})) \right] \quad (16)$$

which can be further simplify and obtains,

$$U^*(Y) = \frac{Gr}{C^2(1-e^{-C})(1-e^{v_0})} \left[ \left( 1 + \frac{Pr}{1-Pr} \right) (e^{-C} - e^{v_0} - e^{v_0 Y} (e^{-C} - 1) - e^{-CY} (1 - e^{v_0})) + (1 - \theta_0(1 - e^{-C})) (C - Pr(1 - e^{v_0}) - C e^{v_0 Y} + (1 - e^{v_0})(Pr - CY)) \right] \quad (17)$$

where,

$$C_3 = \frac{\left( 1 + \frac{Pr}{1-Pr} \right) (e^{-C} - e^{v_0}) + (1 - \theta_0(1 - e^{-C})) (C - Pr(1 - e^{v_0}))}{C^2(1 - e^{-C})(1 - e^{v_0})}, C_4 = -Gr \frac{\left[ \left( 1 + \frac{Pr}{1-Pr} \right) (e^{-C} - 1) + C(1 - \theta_0(1 - e^{-C})) \right]}{C^2(1 - e^{-C})(1 - e^{v_0})}.$$

equation (4) becomes,

$$\frac{\partial U^*}{\partial t^*} - C \frac{\partial U^*}{\partial Y} = Pr \frac{\partial^2 U^*}{\partial Y^2} + PrGr \left( -\theta_0 + \frac{e^C}{1-e^C} (e^{-CY} - 1) + \frac{\text{shint}^*}{\cosh d} e^{(P_1^2 t_{max} - t^*)} \cosh dY \right). \quad (18)$$

solving equation (18) and applying boundary condition (6), we obtain the unsteady state solution for velocity distributions as,

$$U_u^*(Y, t^*) = \frac{U_0 \sinh KY e^{\frac{P_2^2}{2} (t_{max} - t^*) - \frac{1}{2} v_0 (1 - Y)}}{K \cosh K - \frac{C}{2Pr} \sinh K} - \frac{\text{shint}^* e^{P_1^2 (t_{max} - t^*)} \left( \frac{e^{dY}}{1 + P_1^2 + dC} + \frac{e^{dY}}{1 + P_1^2 - dC} \right) - \theta_0 t^* + C \theta_0 Y + \frac{Pr Y^2 \theta_0}{2} + \frac{e^C}{1 - e^C} (t^* - CY - \frac{Pr Y^2}{2})}{2}$$

$$\text{where, } C_5 = 0, C_6 = \frac{U_0 e^{\left( \frac{P_2^2}{2} t_{max} - \frac{1}{2} v_0 \right)}}{K \cosh K - \frac{C}{2Pr} \sinh K}, K = \frac{\sqrt{C^2 - 4Pr P_2^2}}{2Pr} \quad (19)$$

Therefore, the velocity distributions across the vents is,

$$U^*(Y, t^*) = \frac{Gr}{C^2(1-e^{-C})(1-e^{v_0})} \left[ \left( 1 + \frac{Pr}{1-Pr} \right) (e^{-C} - e^{v_0} - e^{v_0 Y} (e^{-C} - 1) - e^{-CY} (1 - e^{v_0})) + (1 - \theta_0(1 - e^{-C})) (C - Pr(1 - e^{v_0}) - C e^{v_0 Y} + (1 - e^{v_0})(Pr - CY)) \right] + \frac{U_0 \sinh KY e^{\frac{P_2^2}{2} (t_{max} - t^*) - \frac{1}{2} v_0 (1 - Y)}}{K \cosh K - \frac{C}{2Pr} \sinh K} - \frac{\text{shint}^* e^{P_1^2 (t_{max} - t^*)} \left( \frac{e^{dY}}{1 + P_1^2 + dC} + \frac{e^{-dY}}{1 + P_1^2 - dC} \right) - \theta_0 t^* + C \theta_0 Y + \frac{Pr Y^2 \theta_0}{2} + \frac{e^C}{1 - e^C} (t^* - CY - \frac{Pr Y^2}{2})}{2} \quad (20)$$

after integrating (20) one will obtain volumetric airflow as,

$$Q^*(Y, t^*) = A_T C_d \left[ \frac{Gr}{C^2(1-e^{-C})(1-e^{v_0})} \left( \left( 1 + \frac{Pr}{1-Pr} \right) (e^{-C} - e^{v_0}) \frac{Y}{2} \right) - (e^{-C} - 1) \frac{e^{v_0 Y}}{v_0} + (1 - e^{v_0}) \frac{e^{\frac{CY}{2}}}{C} + \frac{(e^{-C} - 1)}{v_0} - \frac{(1 - e^{v_0})}{C} \right) + (1 - \theta_0(1 - e^{-C})) \left( -Pr(v_0 + (1 - e^{v_0})) \frac{Y}{2} + Pr e^{v_0 \frac{Y}{2}} + (1 - e^{v_0}) \left( Pr \frac{Y}{2} - C \frac{Y^2}{8} \right) - Pr \right) t^* + \frac{U_0 v_0 e^{\frac{P_2^2}{2} (t_{max} - t^*) - \frac{1}{2} v_0}}{2} \left( \sinh K \frac{Y}{2} + K^2 P_2^2 v_0 \cosh K \frac{Y}{2} \right) e^{-v_0 \frac{Y}{4} - K P_2^2 \frac{v_0}{2}} + \frac{(K \cosh K + \frac{1}{2} v_0 \sinh K) (1 - K^2 (P_2^2 \frac{v_0}{2})^2)}{2} \right]$$

$$\left( \frac{(\cos t^* - \sin t^*) e^{P_1^2 (t_{max} - t^*)}}{2bcoshdP_1^2 \left(1 - \frac{1}{(P_1^2)^2}\right)} \left( \frac{1 - e^{-\frac{d}{2}}}{1 + P_1^2 + dC} + \frac{e^{-\frac{d}{2}} - 1}{1 + P_1^2 - dC} \right) - \theta_0 t^{*2} \frac{Y}{4} + \right. \\ \left. \left( C\theta_0 \frac{Y^2}{8} + Pr\theta_0 \frac{Y^3}{48} \right) t^* + \frac{e^C}{1 - e^C} \left( t^{*2} \frac{Y}{4} - C \frac{Y^2}{8} t^* - Pr \frac{Y^3}{48} t^* \right) \right) \quad (21)$$

By using an idea from elementary physics for

$$\rho_a = \frac{m^*(Y, t^*)}{Q^*(Y, t^*)} \quad (22)$$

Therefore, the mass- transfer is,

$$m^*(Y, t^*) = A_T \rho_a c_d \left[ \frac{Gr}{c^2(1 - e^{-C})(1 - e^{v_0})} \left( \left( 1 + \frac{Pr}{1 - Pr} \right) \left( (e^{-C} - e^{v_0}) \frac{Y}{2} \right) - (e^{-C} - 1) \frac{e^{v_0 \frac{Y}{2}}}{v_0} + (1 - e^{v_0}) \frac{e^{\frac{C}{2}}}{C} + \frac{(e^{-C} - 1)}{v_0} - \frac{(1 - e^{v_0})}{C} \right) + \right. \\ \left. (1 - \theta_0(1 - e^{-C})) \left( -Pr(v_0 + (1 - e^{v_0})) \frac{Y}{2} + Pre^{v_0 \frac{Y}{2}} + (1 - e^{v_0}) \left( Pr \frac{Y}{2} - C \frac{Y^2}{8} \right) - Pr \right) t^* + \right. \\ \left. \frac{U_0 v_0 e^{-\frac{P_2^2}{2} \left( (t_{max} - t^*) + \frac{1}{2} v_0 \right)}}{KcoshK + \frac{1}{2} v_0 sinhK} \left( \frac{sinhK \frac{Y}{2} + K^2 P_2^2 v_0 coshK \frac{Y}{2}}{KcoshK + \frac{1}{2} v_0 sinhK} e^{-v_0 \frac{Y}{4} - K P_2^2 v_0 \frac{Y}{2}} \right) + \right. \\ \left. \frac{(\cos t^* - \sin t^*) e^{P_1^2 (t_{max} - t^*)}}{2bcoshdP_1^2 \left(1 - \frac{1}{(P_1^2)^2}\right)} \left( \frac{1 - e^{-\frac{d}{2}}}{1 + P_1^2 + dC} + \frac{e^{-\frac{d}{2}} - 1}{1 + P_1^2 - dC} \right) - \theta_0 t^{*2} \frac{Y}{4} + \right. \\ \left. \left( C\theta_0 \frac{Y^2}{8} + Pr\theta_0 \frac{Y^3}{48} \right) t^* + \frac{e^C}{1 - e^C} \left( t^{*2} \frac{Y}{4} - C \frac{Y^2}{8} t^* - Pr \frac{Y^3}{48} t^* \right) \right) \quad (23)$$

#### Notations and greek's words

$C_1, C_2, C_3, C_4, C_5, C_6$  Coefficients

$P_1, P_2$  Separation constants

$K, d, C$  Constants

$A_T$  Total area of the openings in non-dimensional

form

$L$  Line scale

$v_0$  Constant indirect velocity of the air

$P$  Air pressure in dimensional form

$x_w$  Constant width of the vents

$y$  Height of the vents in dimensional form

$Y$  Height of the vents in non- dimensional form

$t$  Time in dimensional form

$t^*$  Time in non- dimensional form

$c_d$  discharge coefficient

$U_0$  Constant velocity of air at  $y = 1, t = t_{max}$

$u$  Velocity of air in dimensional form

$U^*$  Velocity profile in non- dimensional form

$U_u^*$  Unsteady velocity profile in non- dimensional form

#### Greek Symbols

$\rho_a$  Ambient density of air

$T_a$  Ambient temperature of air

$\theta_0$  Effective thermal coefficient

$T$  Air temperature in dimensional form

$\Delta T$  Change of air temperature in dimensional form

$T^*$  Temperature profile in non- dimensional form

$\beta$  Coefficient of thermal expansion

$\alpha$  Thermal conductivity ratio

$\nu$  Kinematic viscosity of fluid

#### Non Dimensional Group

$Pr$  Prandtl number

$Gr$  Grashof number

#### Subscript

$w$  width of the vents

### 4.Asymptotic behavior of the results

Asymptotic behavior of the results obtained from equations (13), (20), (21) and (23) are plotted. The analysis of the results is done in order to see the effect of changes of parameters such as involved in the study to the overall flow across the vents while keeping other physical parameters and operating condition fixed.

The paper investigated the transient effect of constant indirect flow velocity in rectangular building with multiple- upper vents induced by stack- driven effect. Although there are three parameters of interest in the present work, effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ) with time interval between  $t^* = 0.00, 0.51$  and  $1.02$ . In this section, the value of  $\theta_0$  used is between  $0.01, 0.03$  and  $0.05$ , since, we assumed the internal heat source is very negligible. Similarly, the value for  $Gr$  is selected arbitrary between  $Gr = 10.0, 20.0$  and  $30.00$  and the value for  $Pr$  is  $Pr = 0.650, 0.710$  and  $0.770$  which is the actual value for air.

Figure 3, 4, 5, 6, 7 and 8 reveal the influence of effective thermal coefficient ( $\theta_0$ ) and Prandtl number( $Pr$ ) on the airflow temperature distributions across the openings. It is clearly seen that airflow temperature increase with the increase of effective thermal coefficient ( $\theta_0$ ) and Prandtl number( $Pr$ ).

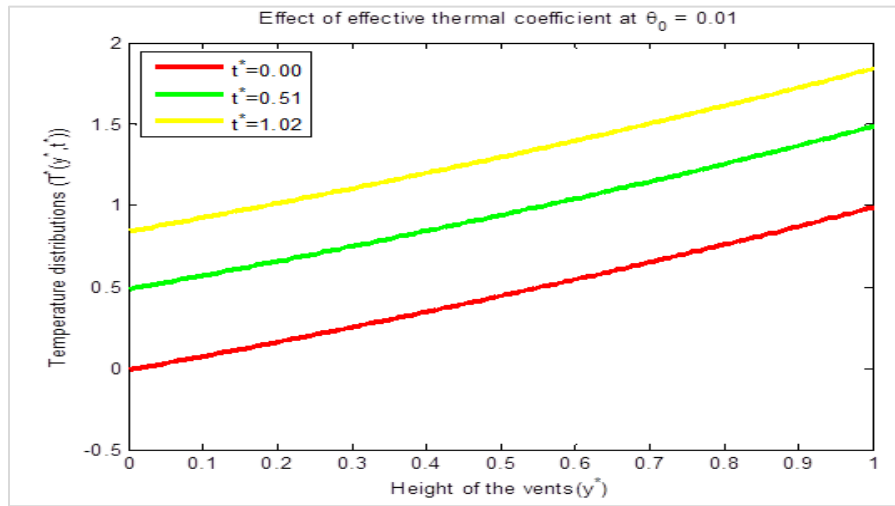
Figure 9, 10, 11, 12, 13, 14, 15, 16 and 17 reveal the influence of effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ) (is a non- dimensional group which approximate the ratio of the buoyancy to viscous force acting on a fluid), on the airflow velocity distributions across the openings. It is clearly seen that airflow velocity increase with the increase of effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ). This is physically true since growing

Prandtl number decreases thermal diffusivity of the air.

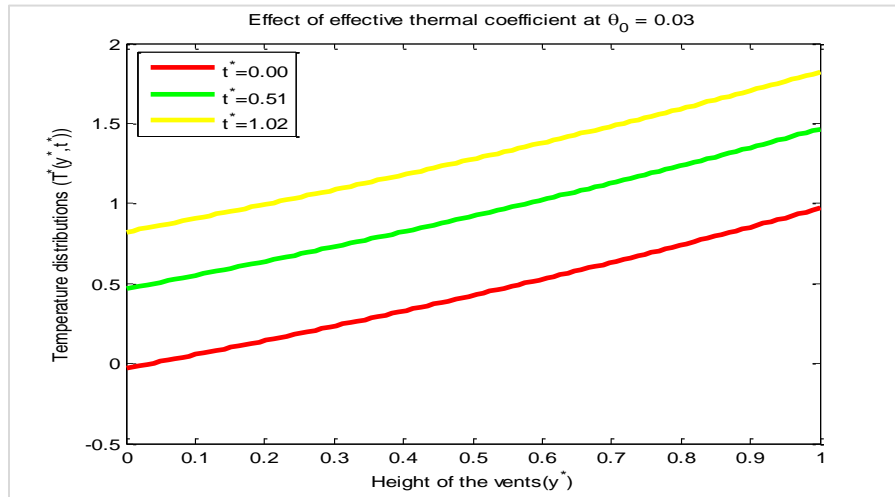
Figure 18, 19, 20, 21, 22, 23, 24, 25 and 26 reveal the influence of effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ) on the volumetric airflow in the building. can be discovered that volumetric airflow goes significantly upward with the increase of effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ). Figure 27, 28, 29, 30, 31, 32, 33, 34 and 35 reveal the influence of effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ) on the mass- transfer. It can be discovered that mass-transfer goes significantly with the increase of

effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ).

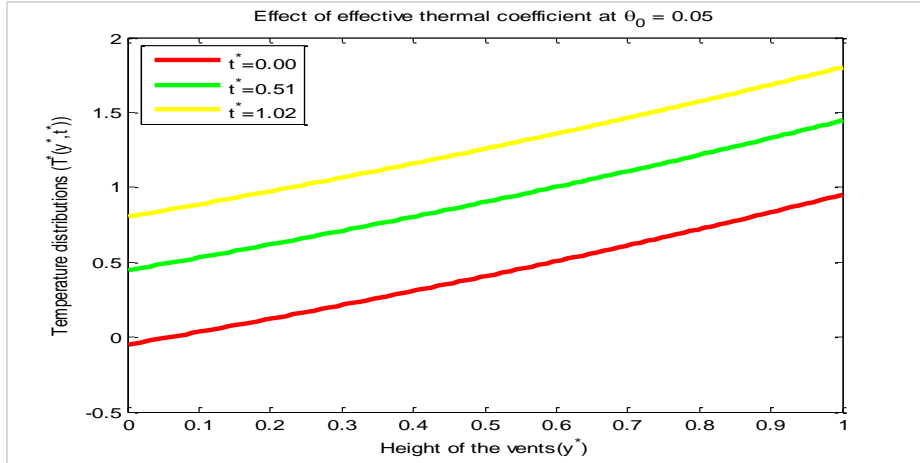
Figure 36, 37 and 38 shows the comparison between present and the previous work by [15]. The main contributions from the present work is that, effect of changes of parameters involved in the results goes significantly upward compared with the previous work. Therefore, effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ) exerts significant influence on the airflow velocity, temperature distributions together with mass- transfer and volumetric airflow in the building envelope.



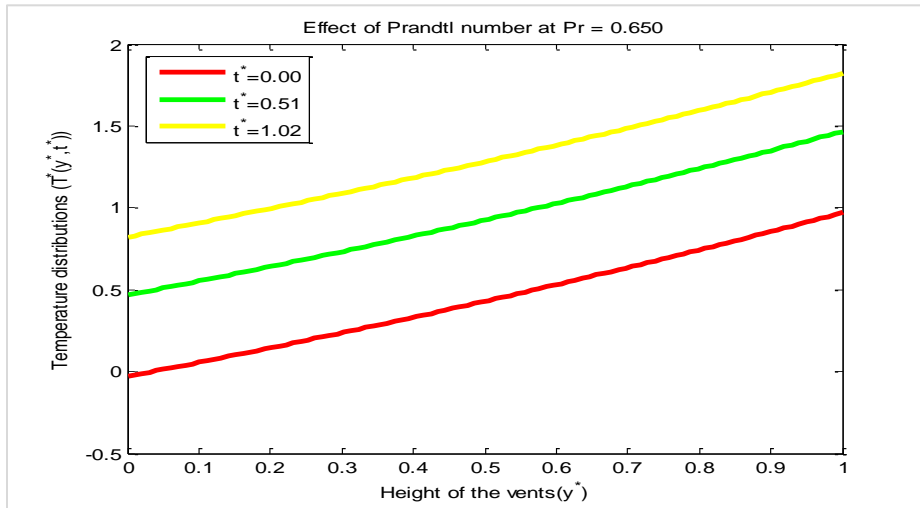
**Figure 3** Airflow temperature distributions for different values of  $t^*$  ( $\theta_0 = 0.01$ )



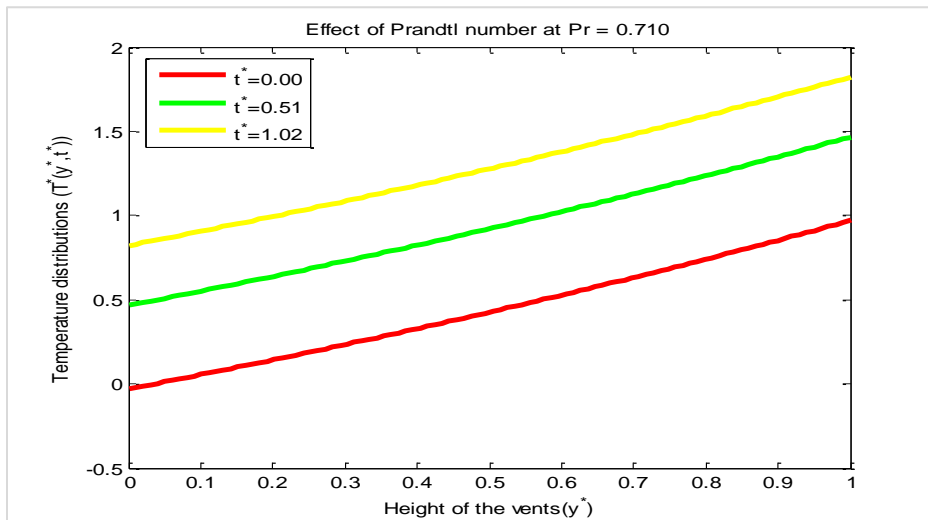
**Figure 4** Airflow temperature distributions for different values of  $t^*$  ( $\theta_0 = 0.03$ )



**Figure 5** Airflow temperature distributions for different values of  $t^*$  ( $\theta_0 = 0.05$ )

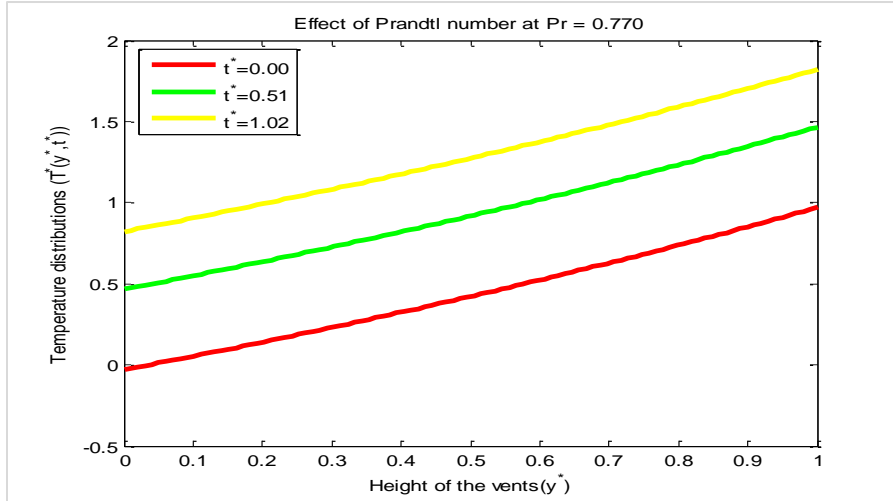


**Figure 6** Airflow temperature distributions for different values of  $t^*$  ( $Pr = 0.650, \theta_0 = 0.03$ )

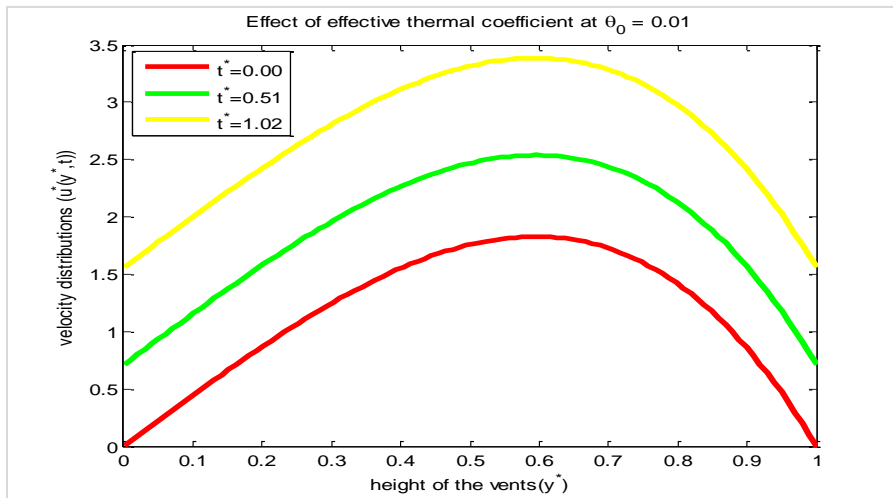


**Figure 7** Airflow temperature distributions for different values of  $t^*$  ( $Pr = 0.710, \theta_0 = 0.03$ )

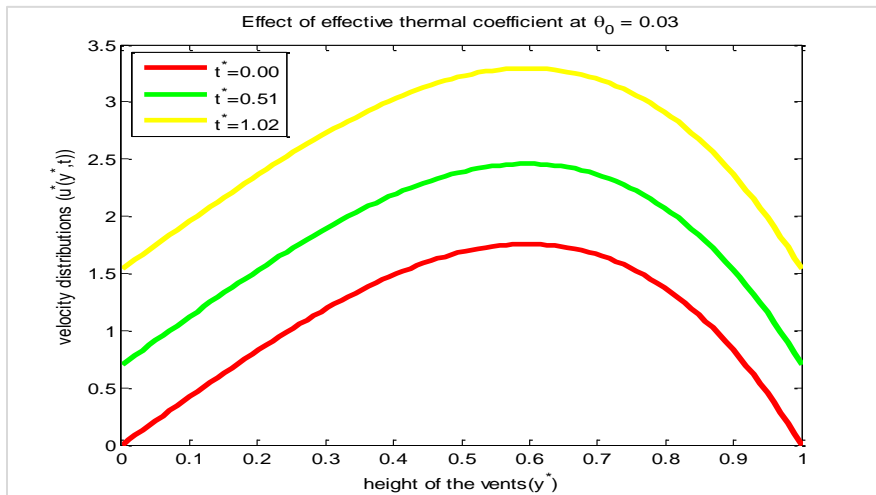




**Figure 8** Airflow temperature distributions for different values of  $t^*$  ( $Pr = 0.770, \theta_0 = 0.03$ )

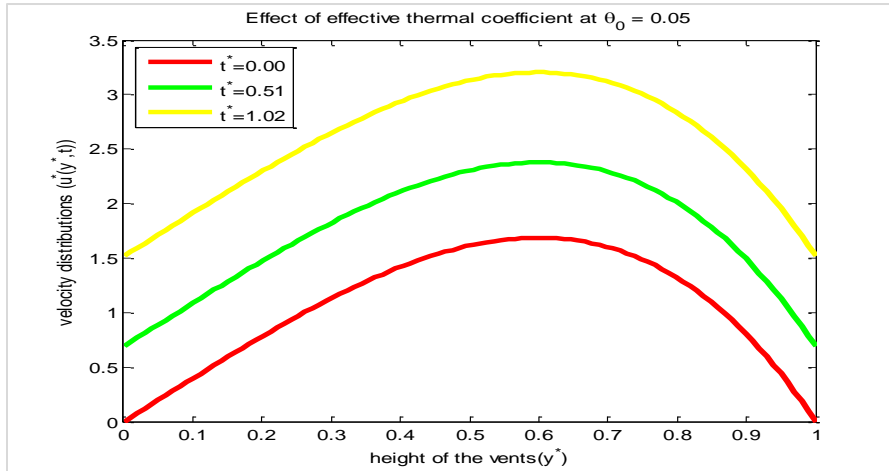


**Figure 9** Airflow velocity distributions for different values of  $t^*$  ( $\theta_0 = 0.01, Gr = 20, Pr = 0.710$ )

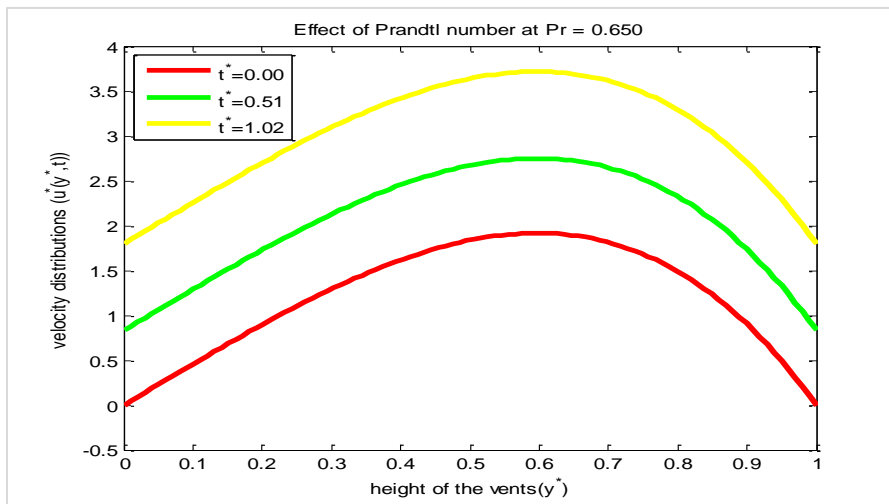


**Figure 10** Airflow velocity distributions for different values of  $t^*$  ( $\theta_0 = 0.03, Gr = 20, Pr = 0.710$ )

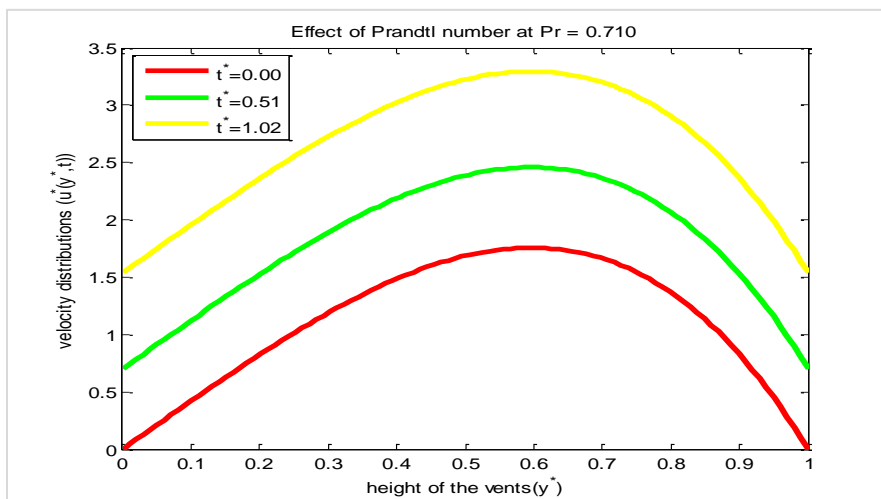




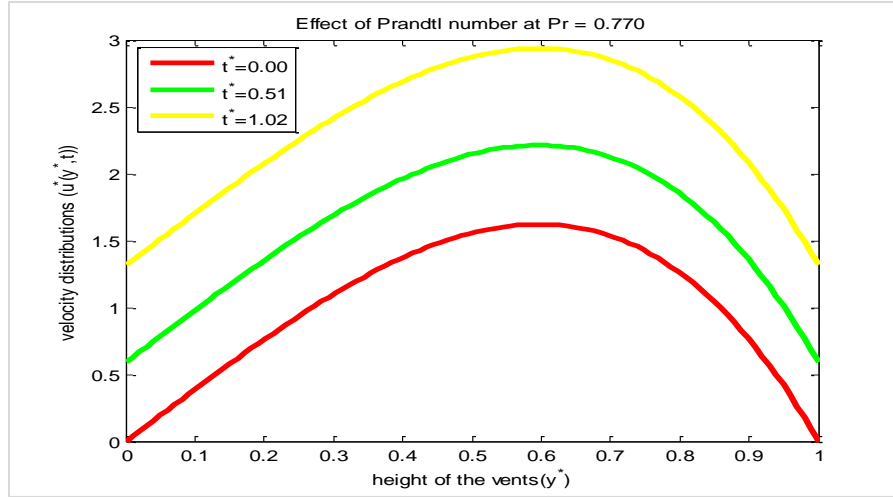
**Figure 11** Airflow velocity distributions for different values of  $t^*$  ( $\theta_0 = 0.05, Gr = 20, Pr = 0.710$ )



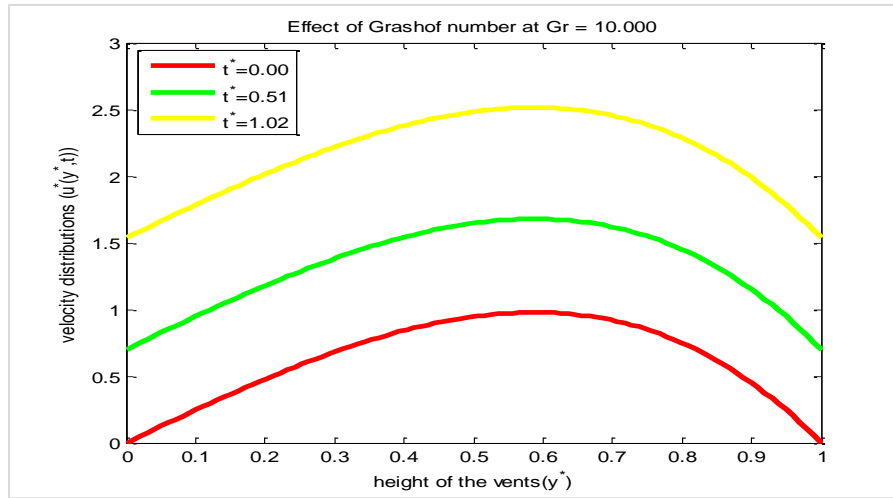
**Figure 12** Airflow velocity distributions for different values of  $t^*$  ( $Pr = 0.650, \theta_0 = 0.03, Gr = 20$ )



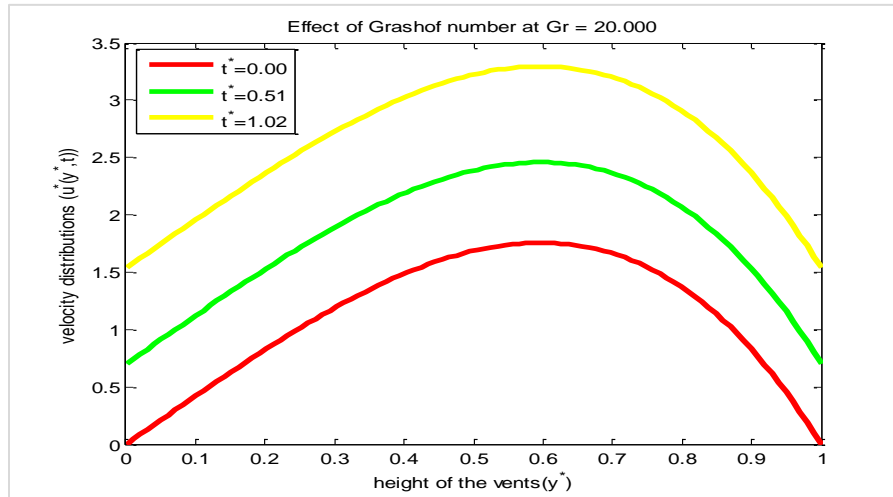
**Figure 13** Airflow velocity distributions for different values of  $t^*$  ( $Pr = 0.710, \theta_0 = 0.03, Gr = 20$ )



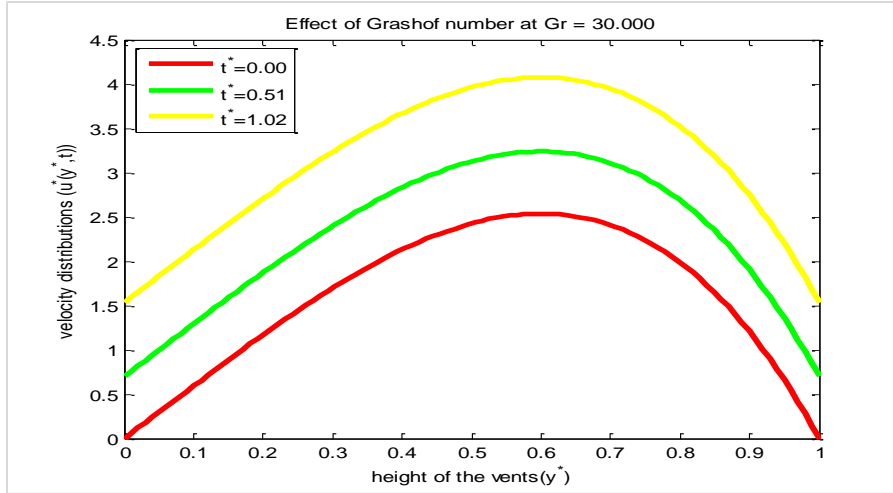
**Figure 14** Airflow velocity distributions for different values of  $t^*$  ( $Pr = 0.770, \theta_0 = 0.03, Gr = 20$ )



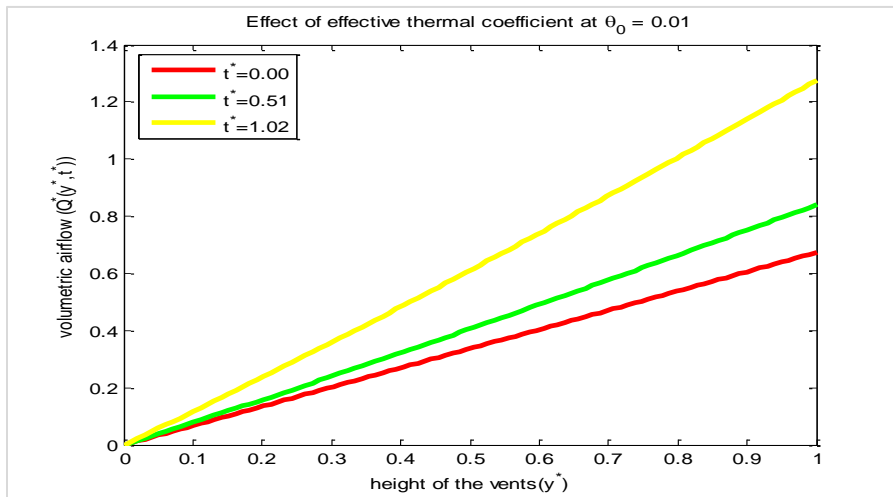
**Figure 15** Airflow velocity distributions for different values of  $t^*$  ( $Gr = 10, Pr = 0.710, \theta_0 = 0.03$ )



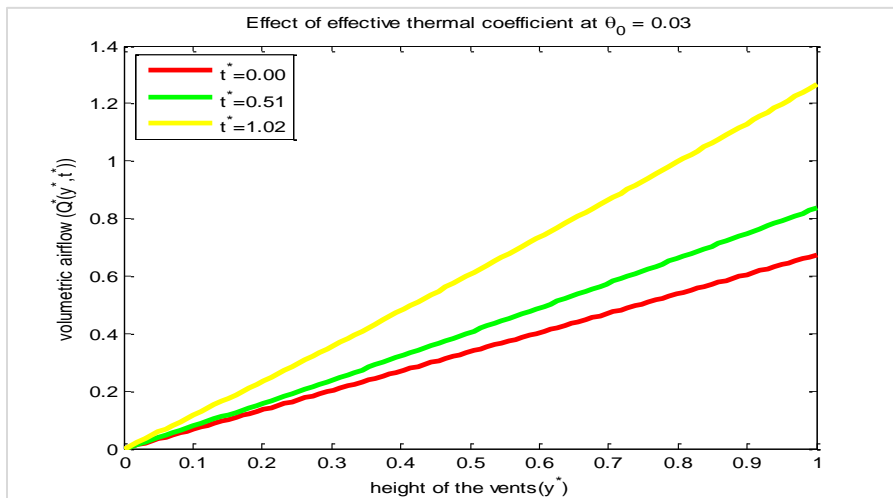
**Figure 16** Airflow velocity distributions for different values of  $t^*$  ( $Gr = 20, Pr = 0.710, \theta_0 = 0.03$ )



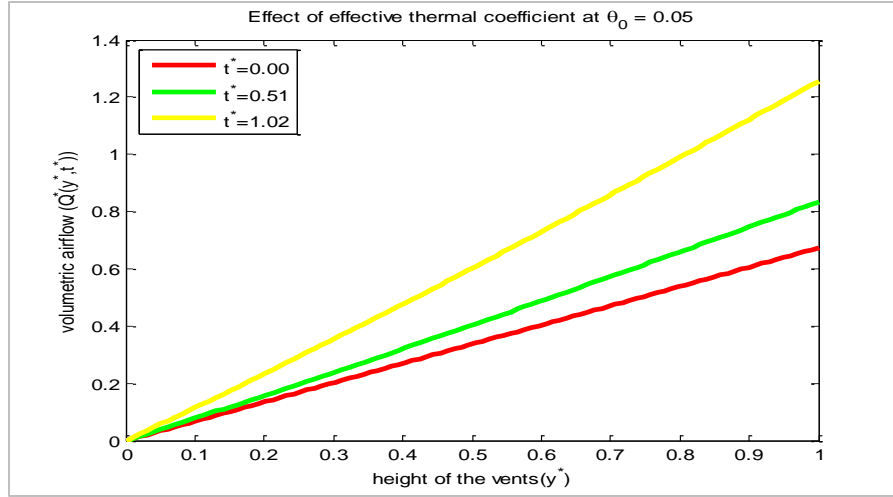
**Figure 17** Airflow velocity distributions for different values of  $t^*$  ( $Gr = 30, Pr = 0.710, \theta_0 = 0.03$ )



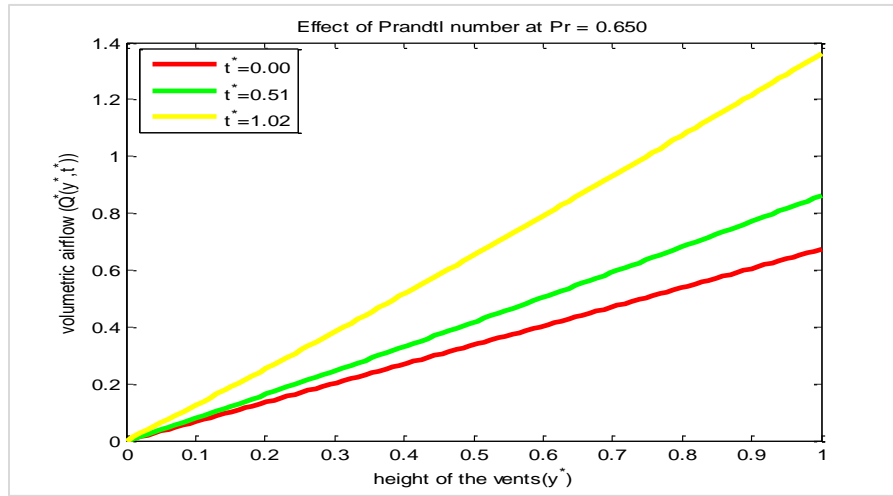
**Figure 18** Volumetric airflow for different values of  $t^*$  ( $\theta_0 = 0.01, Gr = 20, Pr = 0.710$ )



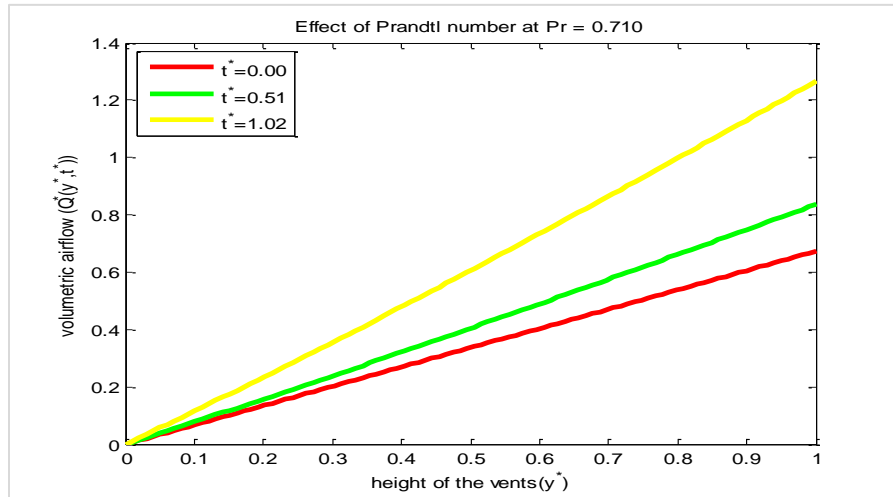
**Figure 19** Volumetric airflow for different values of  $t^*$  ( $\theta_0 = 0.03, Gr = 20, Pr = 0.710$ )



**Figure 20** Volumetric airflow for different values of  $t^*$  ( $\theta_0 = 0.05, Gr = 20, Pr = 0.710$ )



**Figure 21** Volumetric airflow for different values of  $t^*$  ( $Pr = 0.650, \theta_0 = 0.03, Gr = 20$ )



**Figure 22** Volumetric airflow for different values of  $t^*$  ( $Pr = 0.710, \theta_0 = 0.03, Gr = 20$ )

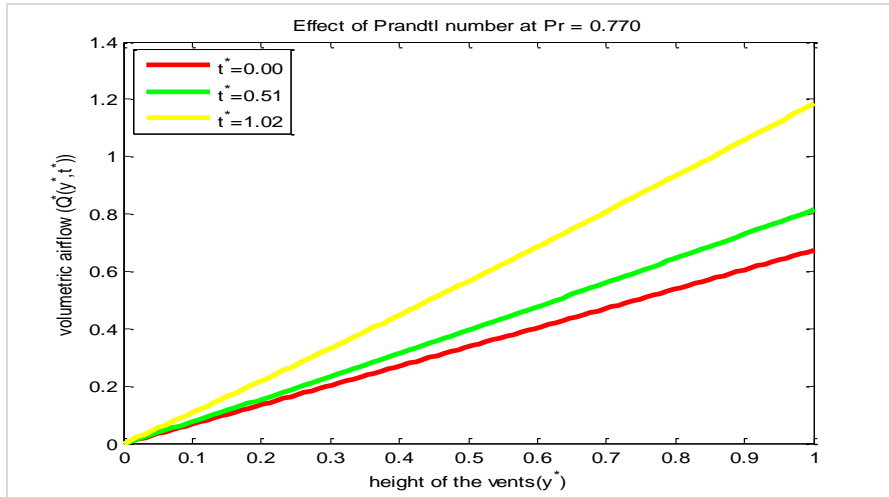


Figure 23 Volumetric airflow for different values of  $t^*$  ( $Pr = 0.770, \theta_0 = 0.03, Gr = 20$ )

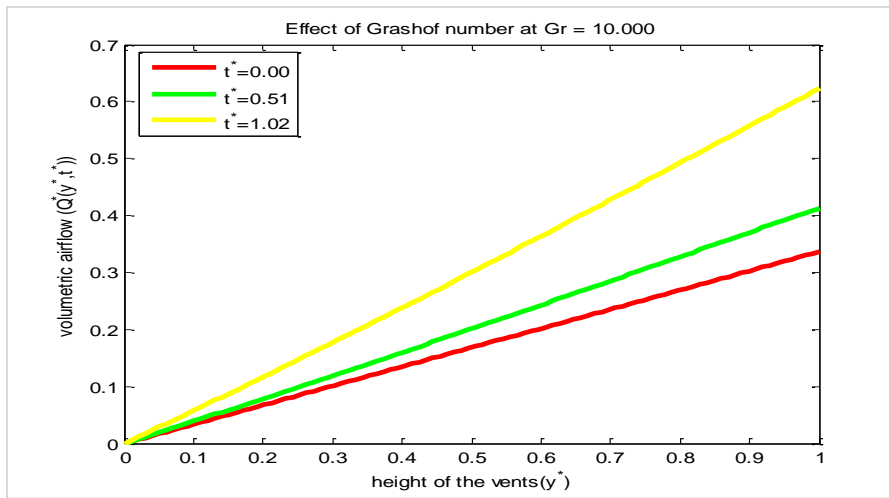


Figure 24 Volumetric airflow for different values of  $t^*$  ( $Gr = 10, Pr = 0.710, \theta_0 = 0.03$ )

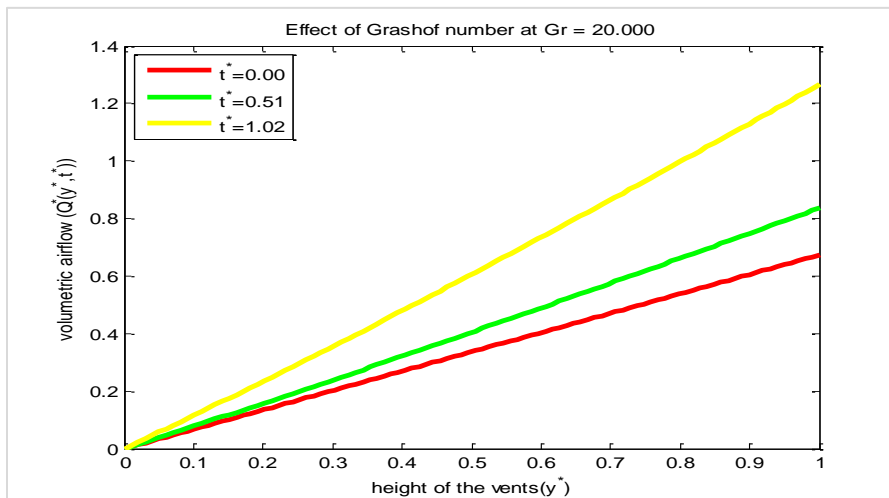
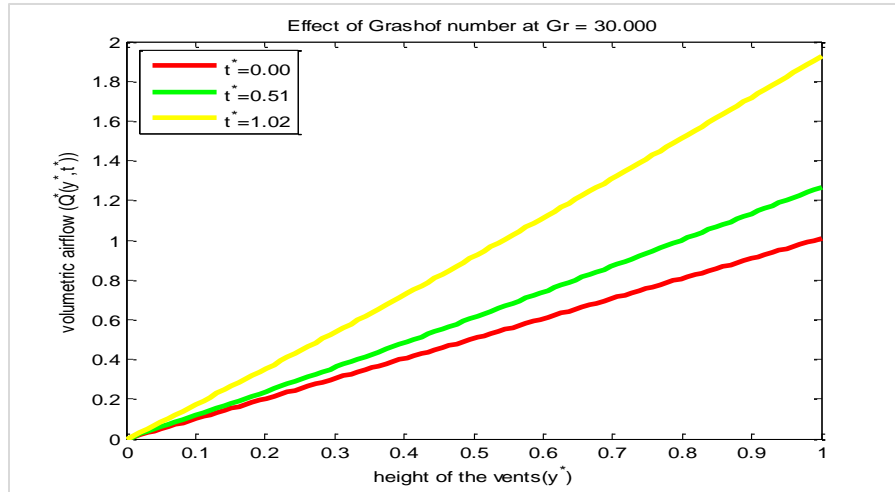
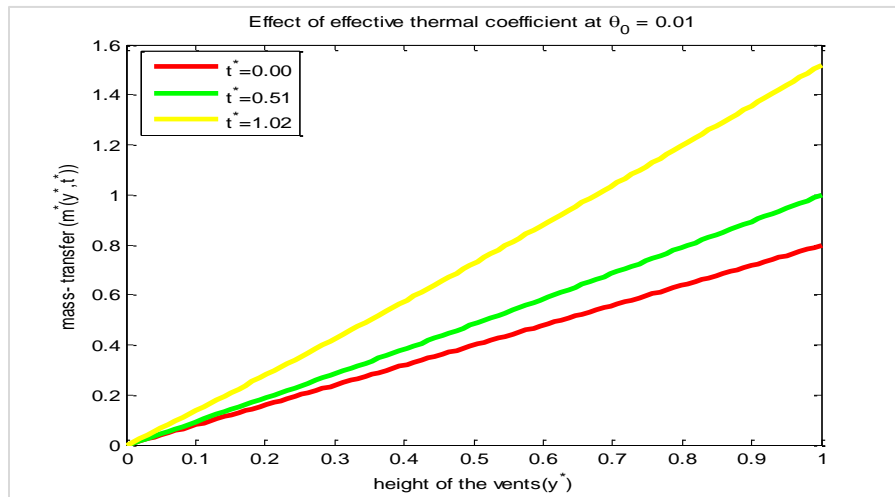


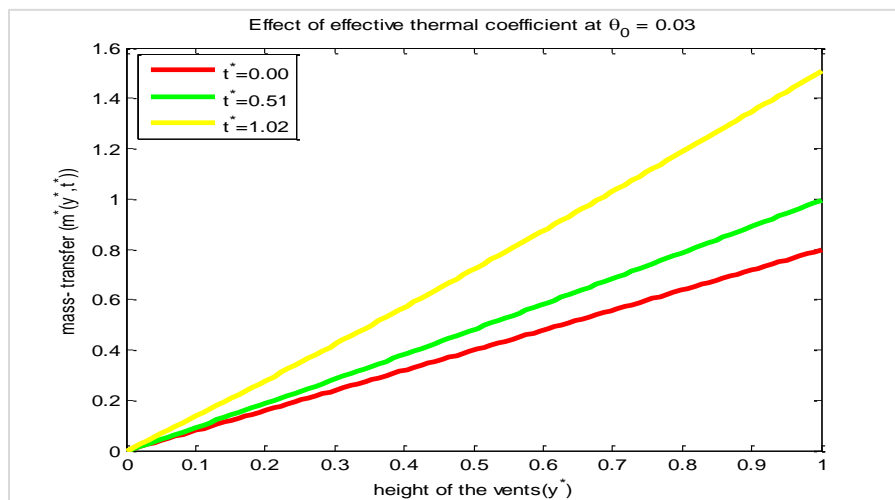
Figure 25 Volumetric airflow for different values of  $t^*$  ( $Gr = 20, Pr = 0.710, \theta_0 = 0.03$ )



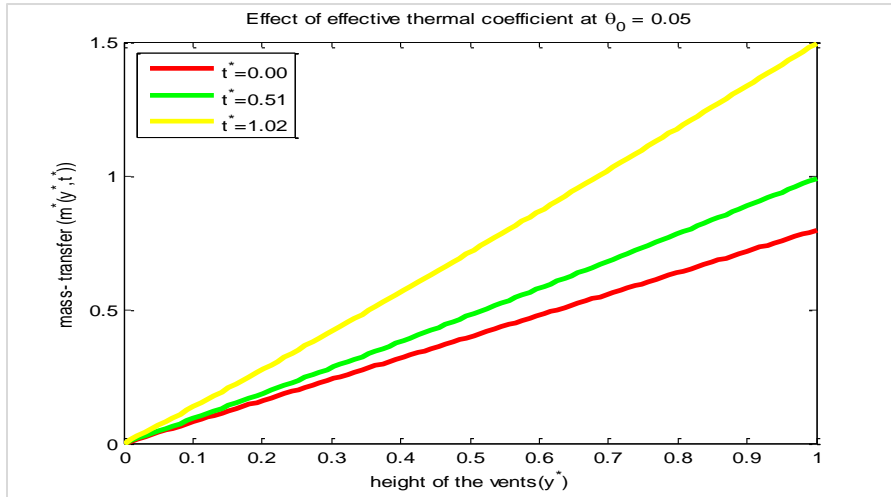
**Figure 26** Volumetric airflow for different values of  $t^*$  ( $Gr = 30, Pr = 0.710, \theta_0 = 0.03$ )



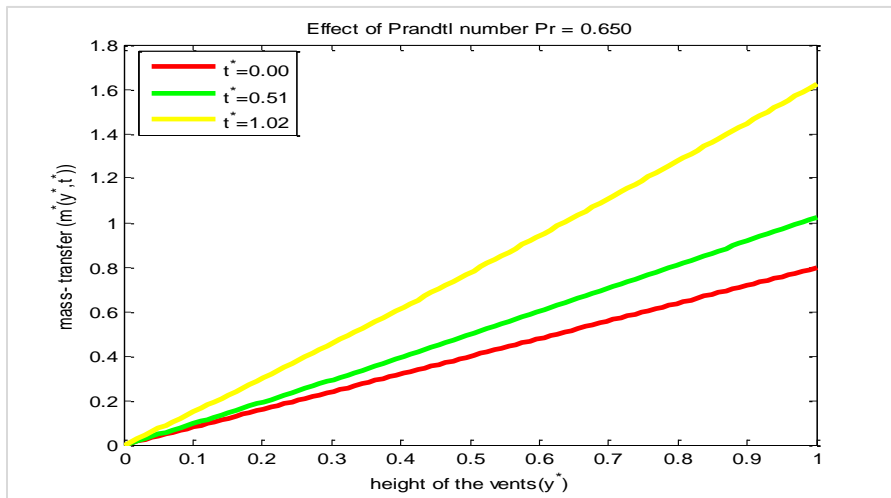
**Figure 27** Mass- transfer for different values of  $t^*$  ( $\theta_0 = 0.01, Gr = 20, Pr = 0.710$ )



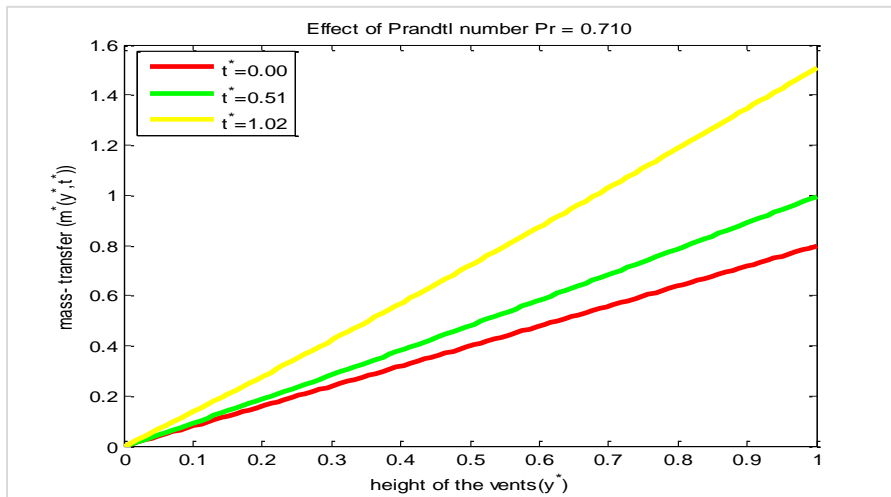
**Figure 28** Mass- transfer for different values of  $t^*$  ( $\theta_0 = 0.03, Gr = 20, Pr = 0.710$ )



**Figure 29** Mass- transfer for different values of  $t^*(\theta_0 = 0.05, Gr = 20, Pr = 0.710)$

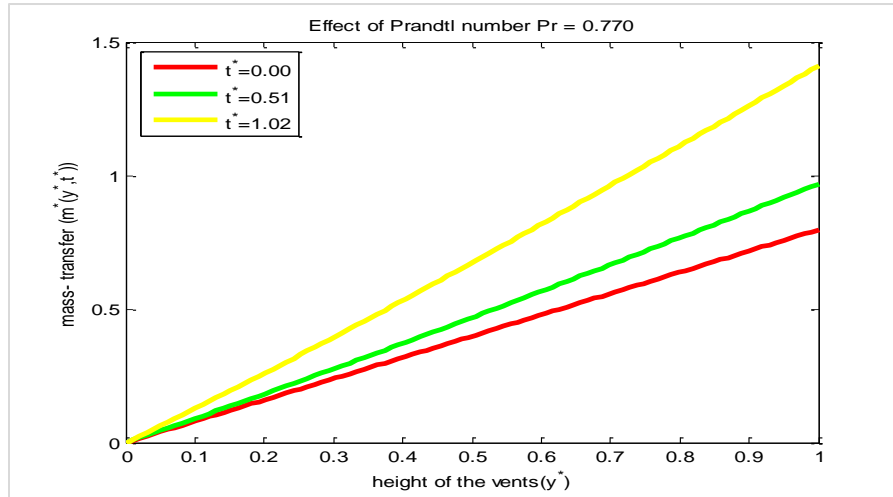


**Figure 30** Mass- transfer for different values of  $t^*(Pr = 0.650, \theta_0 = 0.03, Gr = 20)$

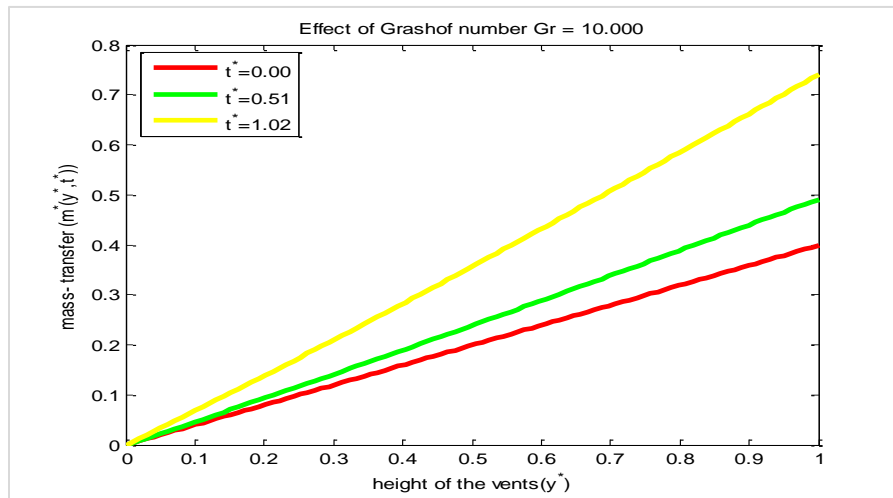


**Figure 31** Mass- transfer for different values of  $t^*(Pr = 0.710, \theta_0 = 0.03, Gr = 20)$

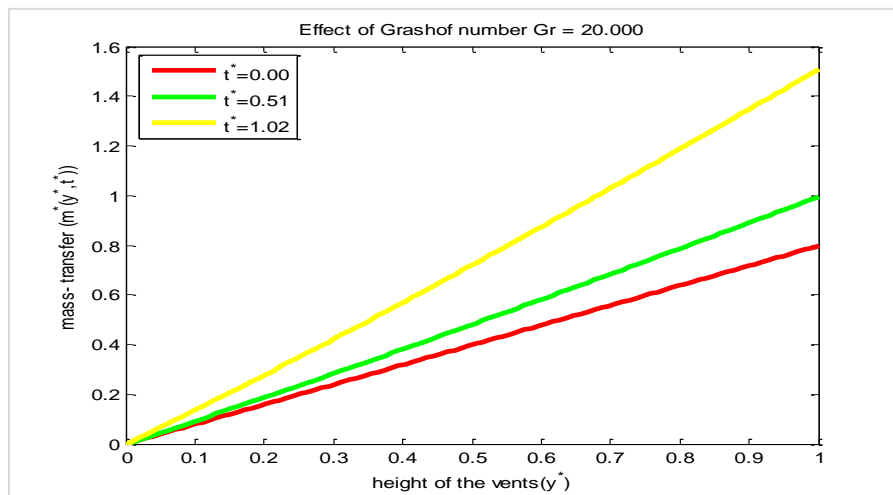




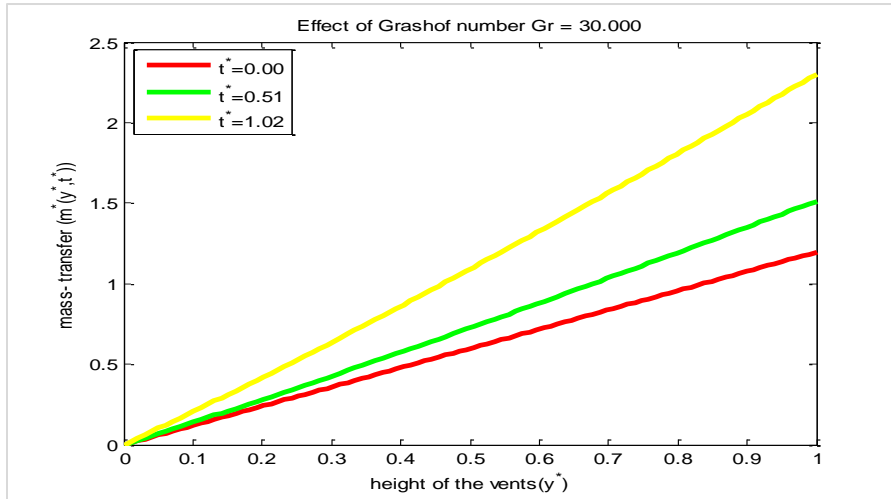
**Figure 32** Mass- transfer for different values of  $t^*$  ( $Pr = 0.770, \theta_0 = 0.03, Gr = 20$ )



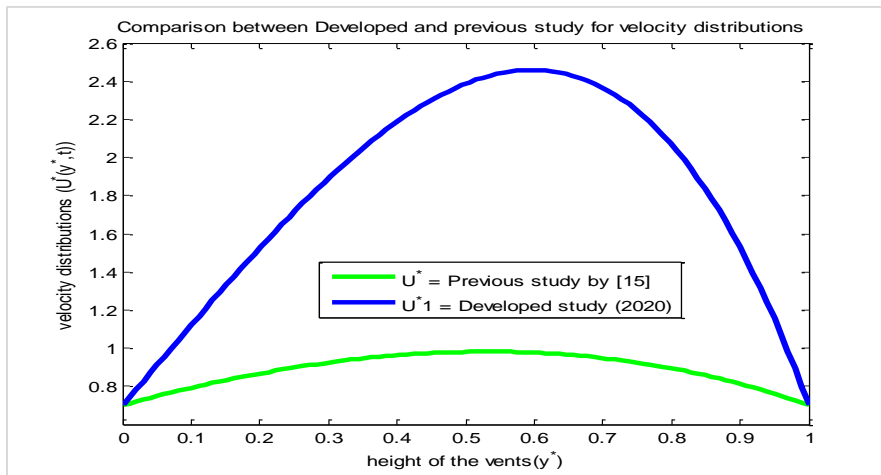
**Figure 33** Mass- transfer for different values of  $t^*$  ( $Gr = 10, \theta_0 = 0.03, Pr = 0.710$ )



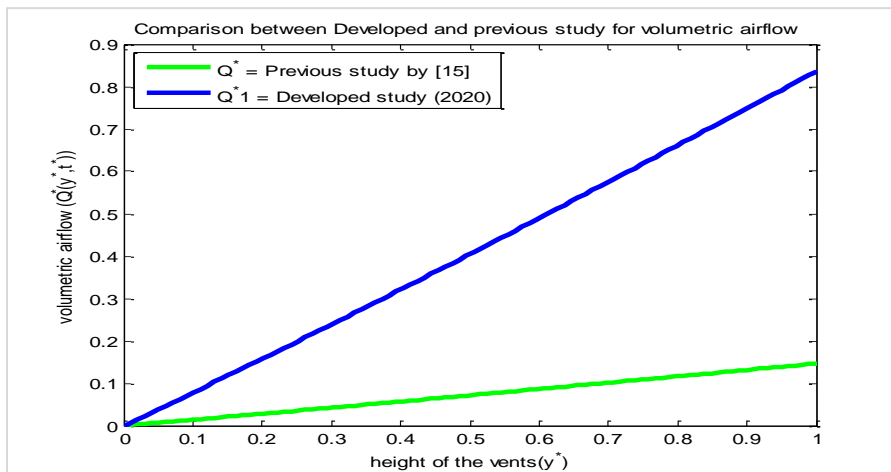
**Figure 34** Mass- transfer for different values of  $t^*$  ( $Gr = 20, \theta_0 = 0.03, Pr = 0.710$ )



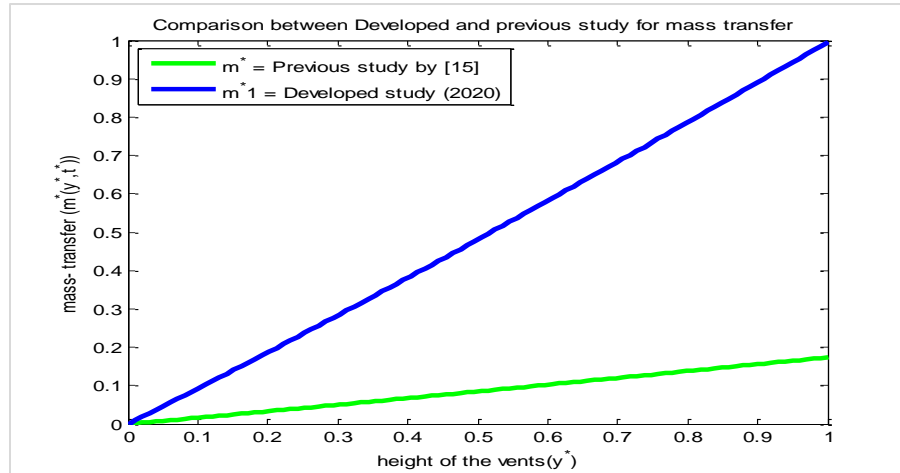
**Figure 35** Mass- transfer for different values of  $t^*$  ( $Gr = 30, \theta_0 = 0.03, Pr = 0.710$ )



**Figure 36** Comparison between velocity profiles  $U^*$  and  $U^*1$  for fixed values of  $\theta_0 = 0.03, Pr = 0.710$ , and  $Gr = 20$



**Figure 37** Comparison between volumetric airflow  $Q^*$  and  $Q^*1$  for fixed values of  $\theta_0 = 0.03, Pr = 0.710$ , and  $Gr = 20$



**Figure 38:** Comparison between mass transfer  $m^*$  and  $m^*1$  for fixed values of  $\theta_0 = 0.03$ ,  $Pr = 0.710$ , and  $Gr = 20$

## 5. Conclusion

In this paper, transient effect of constant indirect flow velocity in rectangular building with multiple- upper vents induced by stack- driven effect was studied. The governing equations describing the flow are written in dimensionless form and solved theoretically by means of separation of variable method. The effect of each physical parameter involved in the study is discussed with aid of graphs. It was found that effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ) exerts significant influence on the airflow velocity, temperature distributions together with mass- transfer and volumetric airflow.

The following major conclusions have been achieved from the paper.

- 1- Temperature increase with an increase of effective thermal coefficient ( $\theta_0$ ) and Prandtl number( $Pr$ ).
- 2- Velocity distributions across the openings increase with an increase of effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ).
- 3- Volumetric airflow increase with an increase of effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ).
- 4- Mass- transfer increase with an increase of effective thermal coefficient ( $\theta_0$ ), Prandtl number( $Pr$ ) and Grashof number( $Gr$ ).
- 5- Present results for velocity distributions, volumetric airflow and mass- transfer goes significantly upward compared with the results for previous intervention given by [15]

## Acknowledgment

None.

## Conflicts of interest

The authors have no conflicts of interest to declare.

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