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Genetic algorithm based optimization for system of nonlinear equations

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Abstract
This work explores a perspective for solving system of nonlinear equations which is an eminent problem in all scientific disciplines. An evolutionary computational technique is used to handle the nonlinear system of equations by converting it into a multi-objective optimization problem. A new fitness function has been proposed, the parameters have been chosen by prior empirical analysis. To validate the performance of the proposed methodology, sensitivity analysis has been carried out by varying the parameters of Genetic algorithm. The results obtained by new approach are quite encouraging and are also compared with other existing work.

Keywords
Evolutionary technique, Genetic algorithm, Nonlinear equations, Optimization.

1.Introduction
Nonlinear optimization problems are quite important and commonly arise in Science and Engineering domains. Various problems, polynomial or transcendental in nature arising in many domains are solved by reducing to nonlinear form. Many numerical methods such Newton-Raphson, projection method etc., are available to derive the solutions for these problems but the convergence of these methods is immensely sensitive to a good initial guess. These methods work efficiently only if the initial start is closer to the solutions of the concerned equation and hence, generally fails. However, these stochastic methods rely on the ground of global search, and thus, have potential to efficiently solve such nonlinear systems, independent of the choice of initial guess. The aim of the present work is to exploit the robustness of evolutionary algorithm via genetic algorithm (GA) to provide a reliable platform to handle these problems. These evolutionary methods have been widely used for getting approximate solutions of nonlinear equations. The detailed related work has been discussed in the next section. These applications were the motivation for the author to further explore the methods to develop an efficient and reliable framework for handling nonlinear and transcendental equations. In the current study, GA [1] has been used for solving non-linear system of equations (NSE).

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The proposed technique is evaluated on two benchmark systems of equations [2−4]. To validate the performance of the proposed scheme, sensitivity analysis by variation in the parameters of GA [5] has been done. A comparative study based on obtained results has also been made to check the convergence of the proposed algorithm.

The major objective of the paper are as follows:
- To explore the application of evolutionary methods in developing an efficient and reliable framework for handling nonlinear and transcendental equations.
- The merit or simply, fitness function for NSE has been formulated carefully as $|f_1(x)| + |f_2(x)| + \cdots + |f_n(x)|$ over previous techniques which used absolutes of sum of the functions $|f_1(x) + f_2(x) + \cdots + f_n(x)|$.
- The benefit of using new fitness function over previous technique returns better results which have been duly compared. Also, it preserves the notion of multi-objective optimization.

The organization of the paper is as follows. Section 2 presents brief related work. Section 3 provides the introductory material on GA algorithm with a flow chart and major contributions of the work is provided. In section 4, the design methodology is given with formulation of fitness function along with parameter selection of GA. In section 5, computational experiments for benchmark problems are presents based on the data generated by independent execution of proposed method. Also, the comparative studies and sensitivity analysis over the variations in parameters is also included. With the last section, major findings, observations are enlisted with future scope and applications.

2.Related work

Bianchini et al. [6], gave an algorithm which confides on special form of gradient descent method. In addition, norm of hessian and norm of gradient are required to have an upper and lower bound respectively. Rovira et al. [7] established a methodology to sort out nonlinear equation to be solved by GA. The methodology tackles the system having large number of variables and where, there is a possibility of many solutions. The work has been verified on system modelling the notion of heat recovery steam generator of a combined cycle power plant at different conditions of load [7]. The work [8] considers parameter estimation of NSE with multi-crossover real coded GA. The efficiency of the method has been compared with other evolutionary techniques by taking unstable plan problem and dead time system. The author, first transformed a system of nonlinear equation into constrained nonlinear programming problem [9], which is then further, solved by sequential quadratic programming algorithms with a line search strategy. Grosan and Abraham in their proposed approach [3], first transform NSE into multi-objective optimization problem and then it is solved by Pareto dominance relation in iterative strategy and solutions that evolve random solutions. The efficiency of the technique has been verified through complex benchmark problems like combustion application, chemical equilibrium application, and kinematic application etc. The author developed a technique for studying the analytical existence of solutions of NSE [10] by applying a matrix inversion principle through Schur complement to a subsystem derived from the main system of equations. For verification of algorithm, an industrial application representing road safety measure has been considered. In [2], the authors proposed an approach by amalgamation of two heuristic approaches, GA and particle swarm optimization (PSO) for solving NSE. For feasibility of the particles travelling in the search space, an additional parameter is introduced, where the algorithm educes the infeasible individuals till they achieve feasibility to arrive at an optimal global solution. The work [11] explored the introduction of some pair of harmonious and symmetric individuals for the production of GA which thereby, ensured diversification in the population. A new elitist model has been introduced to ensure convergence to the solution. In the paper [4], the authors proposed a technique by converting the benchmark problems into constrained and unconstrained optimization problem and then GA has been applied in concurrence with augmented langrangian function. The sensitivity analysis over initial interval and population size has been duly performed. The authors in the work [12], proposed an evolutionary computing technique based on GA. The proposed scheme has been analyzed on a huge set of data generated by variants of GA. Further, mean execution time has been considered and has been verified by statistical analysis of based on 50 independent runs with a different set of initial range on different 20 nonlinear equations in a single variable. In [13], the authors framed a technique in which they first converted the optimization problem into a desirability function as defined by the decision maker in a preferred region. In next phase, the transformed problem is converted into a bi-objective optimization problem, by taking decision maker's satisfaction and hyper-volume as new objectives. The, a set based genetic operators has been defined based on solutions of original problem using simplex method which help in guiding the population towards the desired region of the solution space. The authors in [14] proposed an novel technique for solving NSE through GA by introducing a better fitness function and setting of parameters through elitism to arrive at an approximate solution for the concerned problems.

3.Genetic algorithm

GA is a metaheuristic optimization technique inspired by natural evolution [1−5]. Many computational problems require searching through huge search space. In such cases, GA proves to be efficient and robust tool to arrive at an optimal solution.

GA is based on the survival of the fittest, which combines genetic operations derived from nature. GA provides an efficient way for solving complex nonlinear optimization problems and is a proven tool to generate useful solutions to optimization and search problems [1]. GA initialize with a set of possible solutions where each in GA, is represented as a chromosome. Problem reduction approach helps to map a chromosome with a solution representation. To arrive at an optimum solution, a set of operators are applied to the initial set of solution space. The number of chromosomes in an initial population depends on intuition or empirical analysis. Further iterations invoke various operators like selection, crossover, and mutation to arrive at optimum solution. *Figure 1* depicts the steps involved in solving a problem using GA.

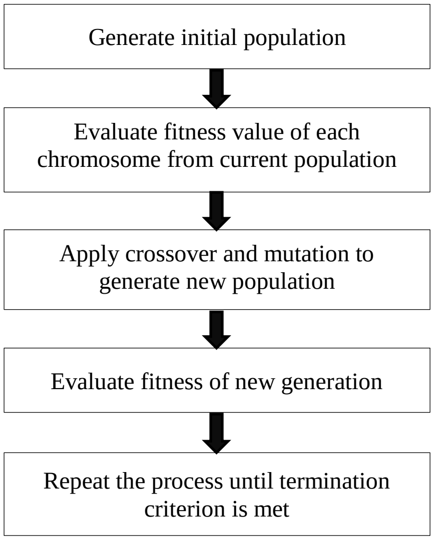

Figure 1 Steps of GA

The stepwise operations used in GA have been described as follows:

3.1Selection

Selection is the process of choosing parents from the population for the application of further operations. It is a method that picks chromosomes out of the initial population, by the evaluation function or function evaluation. Selection can be accomplished using a variety of methods like Roulette wheel selection, Tournament Selection etc. [1].

3.2Crossover

Crossover is the soul of GA, which helps in generating chromosome inheriting features from more than one chromosome. In Crossover, two chromosomes known as parents are combined to form a new chromosome called offspring [5]. The selection of chromosome among the existing chromosomes from the population is done by the defined fitness function. Crossover can be of many types like one point, two point, multi-point, and uniform. The chromosomes produced after crossover, are evaluated for fitness again [1].

3.3Mutation

Mutation plays a vital role in GA. Mutation operator introduces random changes in the characteristic of the chromosomes. Mutation is many times viewed as a background operator to maintain genetic diversity in the population. Mutation modifies some of the building blocks of the genetic structure. Mutation is a proven tool in GA which helps in escaping from local minima's trap and maintain diversity in the population [1].

4.Methodology

With this section, we present the method of optimizing nonlinear system of equations based on GA.

4.1Formulation of fitness function

Under this work, the problem has been organized into multi-objective optimization problem $f_i(x) = 0,\ i = 1,2,\dots,n$, where $x\ and\ f(x)$ may be real, or complex. The fitness or merit function for NSE has been formulated carefully as $\sum_{i=1}^{n} |f_i(x)|$. We claim that choosing the sum of absolutes of the objective functions as $|f_1(x)| + |f_2(x)| + \cdots + |f_n(x)|$ over the previous techniques which used absolutes of sum of the functions $|f_1(x) + f_2(x) + \cdots + f_n(x)|$, is better. The reason in support of this statement is that, if in a problem with two functions $f_1(x)\ and\ f_2(x)$, value obtained by $f_1(x) = -1$ & $f_2(x) = 1$, then we can clearly analyze that both functions $f_1(x)\ and\ f_2(x)$ didn't achieved optimality, also we can see $|f_1(x) + f_2(x)| = 0$ whereas $|f_1(x)| + |f_2(x)| = 2$. This justifies that choosing the fitness function as sum of absolute of the functions gives us better insight of the optimality of the problem under consideration.

4.2Parameter selection for GA

The performance of the proposed methodology has been verified through an experiential analysis, to set the various parameters of GA. The following parameters have been taken care of: population size, generations, selection function, crossover function, mutation function, scaling function. The parameters taken under consideration have been depicted in *Table 1*.

Table 1 Parameters of GA

S. No.	Parameters	Values
1	Population size	200/250/300
2	Generations	50/51/100/150/200
3	Scaling function	Rank
4	Crossover function	Single-point
5	Mutation function	Gaussian
6	Selection function	Roulette wheel

4.3Experimental set up

The performance of the proposed algorithm has been tested on two benchmark problems taken from [3]. The algorithm has been coded in MATLAB 9a and the simulations has run on Intel i3-2350M CPU 2.30

GHz machine with 6 GB inbuilt RAM. The Pseudo code of the proposed algorithm has been depicted in *Figure 2*.

```
Generate initial population;
Evaluate fitness value of individual in population;
Do:
    Select chromosome from parent population;
    Recombine using mutation and crossover to produce
    children;
    Evaluate fitness of new population;
    Replace population with fitter individual;
While a satisfactory result has been obtained.
```

Figure 2 The pseudo code of GA

5.Computational experiments

5.1Results for function 1

The following NSE [3,4] having two variables (Equation 1) has been considered as test problem:

$$f_1(x_1, x_2) = cos(2x_1) - cos(2x_2) - 0.4 \quad (1)$$

$$f_2(x_1, x_2) = 2(x_2 - x_1) + sin(2x_2) - sin(2x_1) - 1.2$$

The above problem has been considered as a multi-objective problem (Equation 2) as:

$$\left.\begin{aligned} \min f_1(x) = 0 \\ min\, f_2(x) = 0 \end{aligned}\right\} \quad (2)$$

Where $x = (x_1, x_2)$, with fitness function $z = |f_1(x)| + |f_2(x)|$.

Table 2 summarizes the results showing the minimum values of fitness function z by the proposed method for tournament and roulette wheel selection function over various generations. In present study, the value of the fitness function is 0.000585 which is sufficiently closer to zero in comparison to other methods [3]. *Figure 3* shows the minimum values of fitness function obtained for different population and various generations.

Table 2 Minimum values of z for Tournament and Roulette wheel selection over different population and generations

Selection	No. of Generations				
	200	150	100	51	50
TM(PS 200)	0.09612	0.02893	0.03672	0.00570	0.00320
RW(PS 200)	0.02891	0.04951	0.04413	0.00329	0.00588
TM(PS 250)	0.017701	0.00779	0.00201	0.00154	0.00129
RW(PS 250)	0.000585	0.000698	0.00144	0.00107	0.00114

TM: Tournament selection function, RW: Roulette wheel selection function, PS: Population size

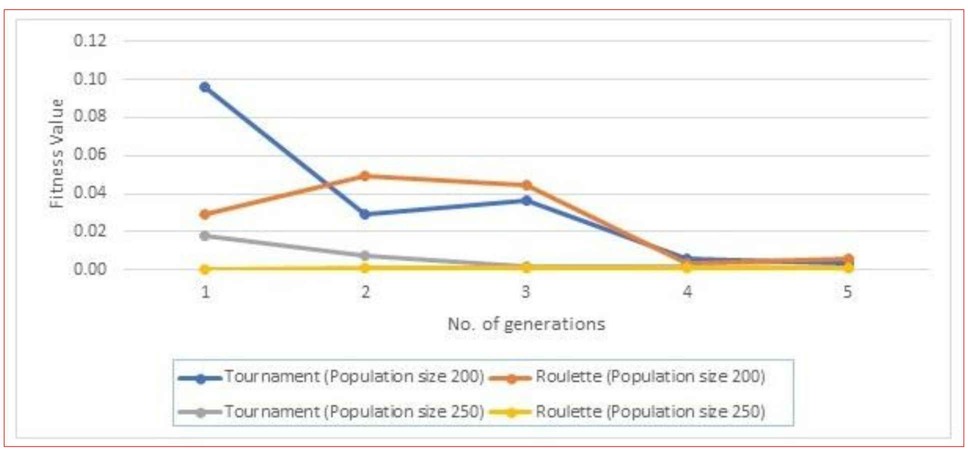

Figure 3 Minimum values of z for tournament and roulette selection over different population and generations

5.1Results for function 2

The following NSE (Equation 3) having two variables has been considered as test problem [2–4]:

$$\left.\begin{aligned} f_1(x_1, x_2) = \cos(2x_1) - \cos(2x_2) - 0.4 \\ f_2(x_1, x_2) = \sin(x_1 x_2) + x_1 + x_2 - 1 \end{aligned}\right\} \quad (3)$$

The above system of equations (Equation 4) has been considered as multi-objective problem as

$$\left.\begin{aligned} \min f_1(x) = 0 \\ min\, f_2(x) = 0 \end{aligned}\right\} \quad (4)$$

Where $x = (x_1, x_2)$, with fitness function $z = |f_1(x)| + |f_2(x)|$.

Table 3 contains the result of system (Equation 4). Furthermore, the comparison showing values of z by the proposed method has been shown in *Figure 4*.

Table 3 Minimum values of z for Tournament and Roulette wheel selection over different population and generations

Selection	No. of Generations				
	200	150	100	51	50
TM(PS 200)	0.00438	0.01229	0.00383	0.00275	0.00201
RW(PS 200)	0.00108	0.00249	0.00246	0.00047	0.00125
TM(PS 300)	0.00555	0.00233	0.00841	0.00194	0.0016
RW(PS 300)	0.000105	0.00159	0.00123	0.00048	0.00036

TM: Tournament selection function, RW: Roulette wheel selection function, PS: Population size

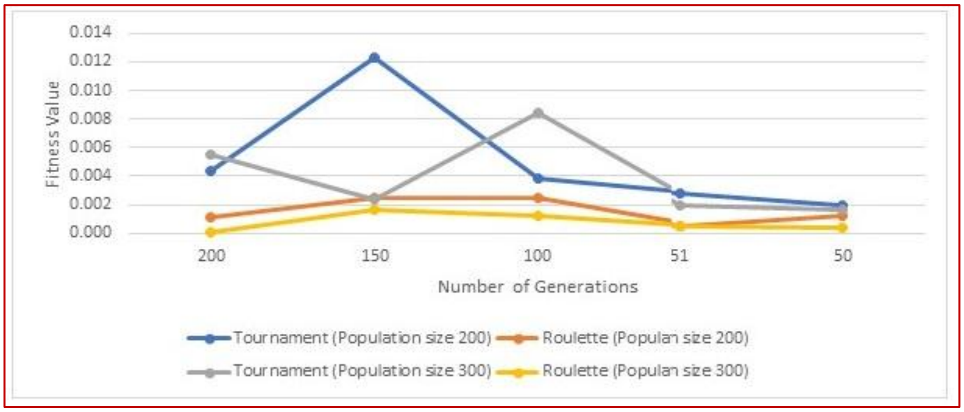

Figure 4 Minimum values of z for Tournament and Roulette selection over different population and generations

The optimal value of fitness function for test problem (5.2) is 0.000105, which is attained at roulette wheel selection function with population size 300 and 200 number of generations.

5.2Sensitivity analysis

The parameters of GA play vital role for its convergence. The scaling function and crossover functions as variable, has been considered for analyzing the proposed algorithm. So, this section presents sensitivity analysis of the proposed algorithm by varying the crossover functions and scaling function for analyzing the chosen parameters in the proposed algorithm.

The variations in the parameters of GA has been presented as follows. The population size and generations has been taken as per the minimum value obtained in the earlier experiment. We observe that in all the problems, optimum value is obtained for Roulette wheel selection function and Gaussian mutation function. Further experiments were carried out by varying crossover function as single point, double point and scattered. Along with this, scaling function has been varied for rank/ proportional/ top/ shift linear/ custom. The ratios were kept as default. The results obtained for new variations in parameters has been Shown in *Table 4 and Table 5* for function 1 and function 2 respectively.

5.3Comparison for functions under consideration

In this subsection, we present the comparison for the values of the functions under consideration. We compare the obtained results for functions by the proposed technique and further variations in the parameters of the proposed technique with the earlier work [3]. The comparison for values has been shown in *Table 6*.

Table 4 Results obtained by variations in parameters for Function 1

Single point crossover				Double point crossover					Scattered				
Proportional	**Top**	**Shift linear**	**Custom**	**Rank**	**Proportional**	**Top**	**Shift linear**	**Custom**	**Rank**	**Proportional**	**Top**	**Shift linear**	**Custom**
0.020943479	#	0.02318996	##	0.00194722	0.00950435	#	0.01737480	0.01572632	0.02518081	0.02251089	#	0.00863991	##
0.013500806		0.02115243		0.00664043	0.01833564		0.01683929	0.01423436	0.03770984	0.01018433		0.01434682	
0.041656965		0.00870357		0.00343553	0.02913464		0.01635381	0.01287384	0.04173928	0.03151282		0.04130295	
0.010632374		0.01946590		0.00990066	0.01167714		0.01583044	0.00565308	0.03345650	0.00811697		0.00972622	
0.024653681		0.01597858		0.01669810	0.01093789		0.02569227	0.01023677	0.03943957	0.02598011		0.010735	
0.005489415		0.00702240		0.02965823	0.02148259		0.01365476	0.01202431	0.03758095	0.03634217		0.02897456	
0.006973995		0.00358295		0.00909452	0.02165700		0.01174642	0.00226371	0.01115393	0.06256584		0.00872280	
0.029162573		0.00652267		0.00899202	0.01035315		0.02458434	0.00585541	0.01303103	0.04545195		0.01589268	
0.010912936		0.00322390		0.01016764	0.00110345		0.01013232	0.00887463	0.04452295	0.02345444		0.01232544	
0.012103252		0.01800819		0.02323087	0.01474728		0.01835674	0.01172737	0.01343389	0.02080175		0.01000375	
0.014503753		0.01480388		0.04472671	0.01757330		0.00883795	0.01832136	0.04535079	0.02691383		0.04289174	
0.029364272		0.00568717		0.02317122	0.02668072		0.00937104	0.00538954	0.01120757	0.01765293		0.03451542	
0.020718221		0.01760744		0.02824460	0.00243071		0.00891401	0.01732477	0.00183576	0.01415816		0.03707848	
0.020078825		0.00518358		0.00266213	0.00542510		0.00823943	0.01562708	0.00293214	0.01889797		0.02308859	
0.027216356		0.01110613		0.03530486	0.00932598		0.01470463	0.02458867	0.01771977	0.02058703		0.04530723	
0.013870855		0.00921210		0.01104677	0.01307307		0.01923159	0.01347032	0.01432550	0.00693097		0.00910373	
0.011601185		0.01411900		0.01844299	0.00663443		0.01074869	0.01126982	0.03053032	0.01537075		0.02284926	
0.014113499		0.0175498		0.00311131	0.02355821		0.00301014	0.02350820	0.01468798	0.02860571		0.02775797	
0.004873881		0.01891783		0.01200310	0.01909033		0.00615697	0.00865531	0.00541285	0.04809935		0.01618069	
0.034149121		0.00461151		0.01433942	0.01398624		0.01985790	0.00309291	0.02878211	0.01053902		0.00766671	
0.025694957		0.01652214		0.01575222	0.00529952		0.03480418	0.00597827	0.00617014	0.02527058		0.00869304	
0.009558075		0.01114478		0.00912619	0.01252574		0.01321959	0.03533052	0.02126871	0.04441523		0.01031355	
0.011013557		0.02581347		0.03374655	0.02405675		0.01275296	0.00717562	0.02604340	0.05481169		0.00458022	
0.021257183		0.01116445		0.00245387	0.01461064		0.02456441	0.01851053	0.04324186	0.01376230		0.01558338	
0.023189965		0.01007440		0.00710504	0.01737480		0.01572632	0.02924183	0.04052780	0.02440166		0.04041848	
Abs 0.018289327		0.01281473		0.01524009	0.01442315		0.01522820	0.01347818	0.02429142	0.02629354		0.02026794	
Min 0.004873881		0.00322390		0.00194722	0.00110345		0.00301014	0.00226371	0.00183576	0.00693097		0.00458022	

Table 5 Results obtained by variations in parameters for Function 2

RW/300/DV/200 (G)													
Single point crossover				Double point crossover					Scattered				
Proportional	**Top**	**Shift linear**	**Custom**	**Rank**	**Proportional**	**Top**	**Shift linear**	**Custom**	**Rank**	**Proportional**	**Top**	**Shift linear**	**Custom**
0.005753008	#	0.001696069	##	0.001025048	0.015496263	#	0.012675672	##	0.014074368	0.017733109	#	0.024135077	##
0.00219487		0.017210892		9.59E-04	0.002775295		0.009014269		0.012045862	0.014831004		0.007511897	
0.012295621		0.017457917		0.001027012	0.005800962		0.029981115		0.018632635	0.010875751		0.008112601	
0.015629331		0.005447637		6.81E-04	0.002556052		0.011278387		0.003273468	0.020923425		0.010023204	
0.005087309		0.019345248		8.44E-04	0.002555703		0.011277089		0.002554999	0.009395689		0.035011611	
0.014186036		0.010376885		0.005969872	0.037613149		0.004762369		0.003442458	0.012939607		0.021583888	
0.007490696		0.003937624		0.005867987	0.026262796		0.016732668		0.004675055	0.001578427		0.033619896	
0.01773647		0.015533394		0.002132547	0.003808165		0.001047255		0.010173392	0.007036875		0.014268854	
0.011018136		0.001043731		0.007078717	0.024201803		0.007898703		0.007885048	0.029820459		0.013673657	
0.046349604		0.009531259		8.41E-04	0.042713383		0.005892275		0.005432727	0.019784798		0.007161397	
0.01652308		0.000219833		0.005465555	0.015186629		0.020864175		0.003557073	0.011842531		0.007945559	
0.005192596		0.012268036		0.004047296	0.018463844		0.002027002		0.003282528	0.014145088		0.016812176	
0.037246374		0.005183543		0.009985248	0.004839556		0.015372102		0.011331567	0.033337794		0.024834373	
0.007161854		0.007815359		5.94E-04	0.016339809		0.022407736		0.001362811	0.010325196		0.004203871	
0.012267483		0.005166691		0.004231998	0.014605661		0.001250818		0.002611945	0.030476501		0.008196192	
0.003349882		0.021379174		0.005330771	0.011167382		0.003403403		0.006157416	0.024979182		0.015627781	
0.011477707		0.017328217		0.003454574	0.001641751		0.009314061		0.017095572	0.003949026		0.013363851	
0.012506418		0.002942704		0.008428261	0.013917095		0.00427691		0.007377098	0.033790868		0.015551547	
0.007782192		0.003597758		0.00916539	0.006526474		0.014662282		0.015535074	0.01804737		0.036166062	
0.025267956		0.004594529		0.002655062	0.030516923		0.017021579		0.011337452	0.009200244		0.008930857	
0.010112212		0.025657317		0.012339041	0.00964344		0.014159102		0.004007261	0.003468754		0.009361058	
0.022820396		0.015298585		0.004648182	0.007265498		0.007081794		0.005492831	0.005865302		0.01116924	
0.004789121		0.01273514		0.003218719	0.023366295		0.01566013		0.001963928	0.007736231		0.010856916	
0.018243274		0.01088283		3.95E-04	0.0053499		0.007038924		0.005347514	0.012602894		0.022478662	
0.001696069		0.012675672		0.00496981	0.006296586		0.015496263		0.004190352	0.024135077		0.012801535	
Avg 0.013367108		0.010373042		0.004214213	0.013956416		0.011223843		0.007313617	0.015552848		0.01573607	
Min 0.001696069		0.000219833		0.000395135	0.001641751		0.001047255		0.001362811	0.001578427		0.004203871	

Table 6 Comparison table

	Earlier work [3]	**Proposed method**	**Varying the parameters**
Function 1	0.0022336	0.000585	0.0011034
Function 2	-0.0028237	0.000105	0.0002198

6.Discussion

Based on experiments and data presented in the last section, the following notions can be drawn:

a) The use of evolutionary technique based on GA for finding the roots of various NSE involving transcendental functions without prior knowledge of initial guess has proved to be an efficient technique.

b) Comparative studies with the earlier works validate the worth of the designed method.

c) The results obtained by the proposed algorithm have been reported in *Table 6*. For function 1, optimal value obtained is 0.000585, which is better than 0.0022336 obtained by [3]. Algorithm helped in improving the result by reducing the error by 0.0016486. Similar observation we can make for function 2, proposed method gave optimal value 0.000105 in comparison to [3] which obtained -0.0028237 and hence, the proposed algorithm helped in improving the result by 10^{-1}.

d) Moreover, we can see the value obtained by varying the parameters through sensitivity analysis also gives an improved value but not better than the actual value obtained by the proposed method.

e) The sensitivity analyses done by further varying the set parameters of GA, the results are worth noticeable as it validates the parameters selection of our algorithm and ensure its application for more complex mathematical problems.

7.Conclusion and future work

This paper proposes a scheme to handle complex system of equations pertaining to various engineering domains. In this work, GA has been used to investigate the solution of such problems. The present work is an extension of our earlier work in which GA has been applied for solving transcendental and nonlinear equations [14]. The convergence of GA depends on its parameters. Initially, the fitness function which is one of the important factor, has been taken as sum of the modulus of the individual function which has not been considered in earlier works. Afterwards, population size, mutation function, crossover function and number of generations have been chosen with empirical analysis. To validate the methodology, sensitivity analysis has also been carried out by varying the crossover functions and scaling functions. The experimentation was carried out well and results have been reported. Finally, the results obtained have been compared with the earlier works [3] and results obtained by varying the parameters have been reported in *Table 6*. It shows that the proposed methodology gives better results. The sensitivity analysis justifies the worth of chosen parameters and hence, the simplicity of the concept along with easy implementation paves way for the application of the proposed work to handle more complex problems involving large set of variables.

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Conflicts of interest

The authors have no conflicts of interest to declare.

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