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Square sum labeling for some lilly related graphs

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Abstract
Let G be a graph is said to be a square sum labeling if there exist a bijection $f : V(G) \to \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^* : E(G) \to N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$, for every $uv \in E(G)$ are all distinct. A graph which admits square sum labeling is called a square sum graph. In this paper we investigate square sum labeling for some graphs related to lilly graph. We discuss square sum labeling in the context of some graph operations namely duplication, fusion, switching in the lilly graph I_n. We prove that the duplication of an apex vertex and other vertex of lilly graph are square sum graph. We prove that the identifying any vertex of lilly graph are square sum graph. We prove that the switching of an apex vertex and other vertex of lilly graph are square sum graph and also apply coloring of the graph operations of the lilly graph is satisfying coloring condition.

Keywords
Square sum labeling, Square sum graph, Lilly graph, Duplication, Fusion, Switching, Coloring.

1.Introduction
In this paper, we consider only simple, finite, undirected and non – trivial graph $G = (V(G), E(G))$ with the vertex set $V(G)$ and the edge set $E(G)$. For all other terminology and notations in graph theory I follow West [3]. If the vertices of the graph are assigned values subject to certain conditions is known as *graph labeling*. A dynamic survey on graph labeling is regularly updated by Gallian [5] and it is published by Electronic Journal of Combinatory. In [9] A. Edward Samuel and S. Kalaivani have proved that prime labeling for some octopus related graphs. In [10] A. Edward Samuel and S. Kalaivani have proved that prime labeling for some planter related graphs. In [11] A. Edward Samuel and S. Kalaivani have proved that prime labeling for some Vanessa related graphs. In [8] Germina K.A and Reena Sebastian have proved that On Square sum graphs. In [4] Ajitha .V, Arumugam .S and Germina .K.A have proved that On Square sum graphs. In [2] Frank Harary have proved some results on Graph theory. Vast amount of literature is available on different types of graph labeling and more than 1000 research papers have been published so far in past three decades.

For any labeling problems following three characteristics are really note-worthy : A set of numbers from which vertex labels are chosen; A rule that assigns a value to each edge; A condition that these values must satisfy. The present work is aimed to discuss one such a labeling namely square sum labeling.

2.Preliminary definitions
Definition [7]
Let $G = (V(G), E(G))$ be a graph is said to be a *square sum labeling* if there exist a bijection $f : V(G) \to \{0, 1, 2, \dots, p-1\}$ such that the induced function $f^* : E(G) \to N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$, for every $uv \in E(G)$ are all distinct. A graph which admits square sum labeling is called a *square sum graph*.

Definition [6]
Duplication of a vertex v_k of a graph G produces a new graph G_1 by adding a vertex v_k' with $N(v_k') = N(v_k)$. In other words a vertex v_k' is said to be a duplication of v_k if all the vertices which are adjacent to v_k are now adjacent to v_k' also.

Definition [6]
Let *u* and *v* be two distinct vertices of a graph G. A new graph G_1 is constructed by *identifying(fusing)* two vertices *u* and *v* by a single vertex *x* is such that

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every edge which was incident with either *u* or *v* in G is now incident with *x* in G_1.

Definition [6]

A *vertex switching* G_v of a graph G is obtained by taking a vertex *v* of G, removing all the entire edges incident with *v* and adding edges joining *v* to every vertex which are not adjacent to *v* in G.

Definition [1]

A k – coloring of a graph $G = (V, E)$ is a function $c : V \to C$, where $|c| = k$. (Most often we use $c = [k]$). Vertices of the same color form a color class. A coloring is *proper* if adjacent vertices have different colors. A graph is *k – colorable* if there is a proper *k* – coloring. The chromatic number $\chi(G)$ of a graph G is the minimum *k* such that G is *k* – colorable.

3.Square sum labeling for some lilly related graphs

3.1Lilly graph

The *Lilly graph* $I_n, n \geq 2$ can be constructed by two star graphs $2K_{1,n}, n \geq 2$ joining two path graphs $2P_n, n \geq 2$ with sharing a common vertex. i.e.,$I_n = 2K_{1,n} + 2P_n$.

In *Figure 1*, represents the lilly graph. If $n = 4$, i. e ., $I_4 = 2K_{1,4} + 2P_4$.

Example 3.2

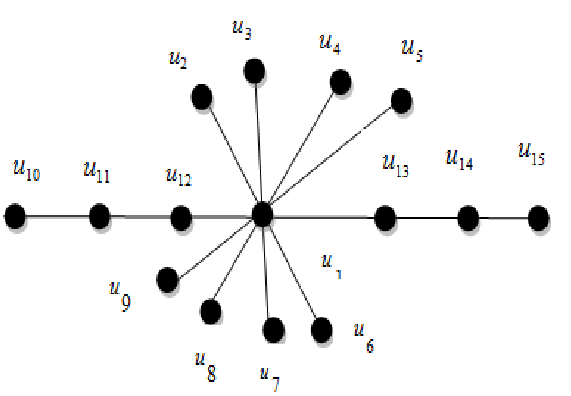

Figure 1 The lilly graph I_4

Theorem 3.3. The lilly graph $I_n, (n \geq 2)$ admits square sum labeling, where *n* is any positive integer.

Proof. Let G be the lilly graph $I_n, (n \geq 2)$. Let $\{u_1, u_2, \ldots, u_{4n-1}\}$ be the vertices of I_n. Here $|V(I_n)| = 4n - 1$. Define a labeling $f : V(I_n) \to \{0, 1, 2, \ldots, 4n - 2\}$ as follows

$$f(u_i) = i - 1 \quad \text{for } 1 \leq i \leq 4n - 1$$

Clearly vertex labels are distinct. And f induces a function $f^* : E(G) \to N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$. Then for any edge $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. Thus f admits square sum labeling. Therefore I_n is a square sum graph.

In *Figure 2* represents the lilly graph satisfies the conditions of prime labeling and coloring with respect to the above theorem 3. 3.

Example 3.4

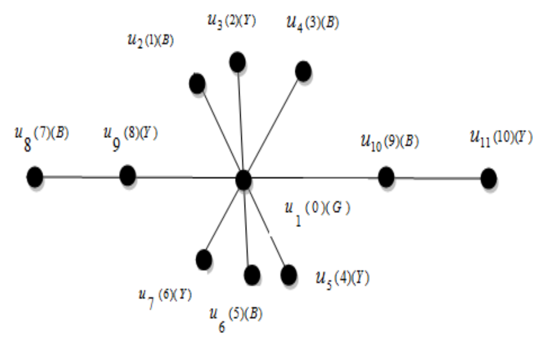

Figure 2 Square Sum labeling for I_3

Theorem 3.5. The graph obtained by duplication of an apex vertex u_1 in the lilly graph $I_n, (n \geq 2)$ is a square sum graph, where *n* is any positive integer.

Proof. Let G be the lilly graph I_n. Let u_1 be an apex vertex of the lilly graph I_n, u_1' be its duplicated of an apex vertex and G_k be the graph resulted due to duplication of an apex vertex u_1 in I_n, where *n* is any positive integer. Let $V(I_n) = \{u_1, u_2, \ldots, u_{4n-1}\}$. Here $|V(G_k)| = 4n$.

We define a labeling $f : V(G_k) \to \{0, 1, 2, \ldots, 4n - 1\}$ as follows.

$$f(u_i) = i - 1 \qquad \text{for } 1 \leq i \leq 4n - 1$$

$$f(u_1') = 4n - 1$$

Clearly vertex labels are distinct. And f induces a function $f^* : E(G) \to N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$. Then for any edge $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. The weights of all the edges of the graph G are distinct. Hence f admits square sum labeling. Therefore I_n is a square sum graph.

In this *Figure 3*, represents the duplication of an apex vertex u_1 in the lilly graph $I_4, (n \geq 2)$ satisfies the prime labeling and coloring conditions with respect to the above theorem 3.5.

Example 3.6

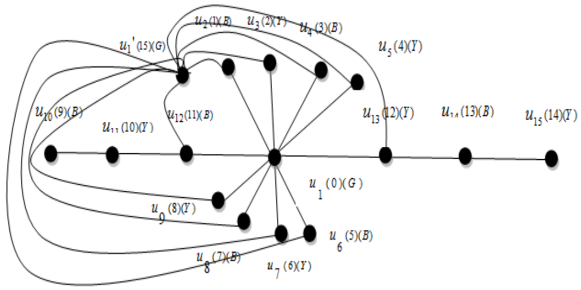

Figure 3 Duplication of an apex vertex $u_1$in I_4

Theorem 3.7. The graph obtained by duplicating of any vertex u_k to $u_k{}'$ in the lilly graph $I_n, (n \geq 2)$ is a square sum graph, where *n* is any positive integer.

Proof. Let G be the lilly graph I_n. Let u_k be any vertex of the lilly graph I_n, $u_k{}'$ be its duplicated of any vertex and G_k be the graph resulted due to duplication of any vertex u_k in I_n, where *n* is any positive integer. Let $V(I_n) = \{u_1, u_2, \dots, u_{4n-1}\}$. Here $|V(G_k)| = 4n$. The mapping $f : V(G_k) \rightarrow \{0, 1, 2, \dots, 4n-1\}$ by

$$f(u_i) = i - 1 \quad \text{for } 1 \leq i \leq 4n-1$$
$$f(u_i{}') = 4n - 1$$

Clearly vertex labels are distinct. And f induces a function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$. Then for any edge $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. The weights of all the edges of the graph G are distinct. Hence f admits square sum labeling. Therefore I_n is a square sum graph.

In this *Figure 4*, represents the duplication of any vertex u_{11} in the lilly graph $I_4, (n \geq 2)$ satisfies the prime labeling and coloring conditions with respect to the above theorem 3.7.

Example 3.8

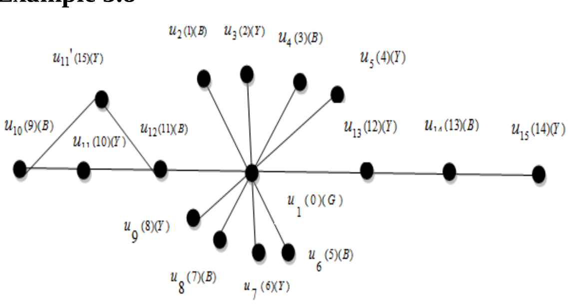

Figure 4 Duplication of any vertex u_{11}in I_4

Theorem 3.9. The graph obtained by identifying any two vertices u_i and u_k (where $d(u_i, u_k) \geq 3$) of the lilly graph $I_n, (n \geq 2)$ is a square sum graph, where *n* is any positive integer.

Proof. Let $I_n, (n \geq 2)$ be the lilly graph with vertices $\{u_1, u_2, \dots, u_{4n-1}\}$ and the vertex u_i be fused with u_k. Denote the resultant graph as G_k. Here we note that $|V(G_k)| = 4n - 2$.The mapping $f: V(G_k) \rightarrow \{0, 1,2, \dots ,4n-3\}$ is defined by

$$f(u_1 = u_{16}) = i - 1 \quad \text{for } i = 1$$
$$f(u_i) = i - 1 \quad \text{for } 2 \leq i \leq 4n-5$$
$$f(u_i) = i \quad \text{for } 4n-3 \leq i \leq 4n-1$$

Clearly vertex labels are distinct. Then f induces a function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$ and for any edge $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. The weights of all the edges of the graph G are distinct. Hence f admits square sum labeling. Therefore I_n is a square sum graph.

In this *Figure 5*, represents the identifying of any two vertices u_1 and u_{16} in the lilly graph $I_5, (n \geq 2)$ satisfies the prime labeling and coloring conditions with respect to the above theorem 3.9.

Example 3.10

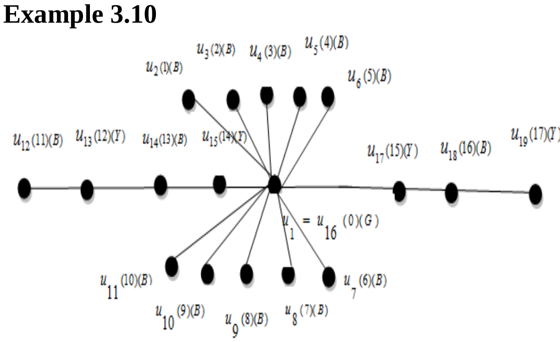

Figure 5 Fusion of u_1 and u_{16} in I_5.

Theorem 3.11. The switching of any vertex u_k in a lilly graph $I_n, (n \geq 2)$ produces a square sum graph, where *n* is any positive integer.

Proof. Let $G = I_n$ and $u_1, u_2, \dots, u_{4n-1}$ be the successive vertices of lilly graph $I_n, (n \geq 2)$ and G_u denotes the graph obtained by a vertex switching of G with respect to the vertex u. It is obvious that $|V(G_u)| = 4n - 1$.Without loss of generality we initiate the labeling from u_1 and proceed in the clock-wise direction. Define a labeling $f: V(G_u) \rightarrow \{0, 1,2, \dots ,4n-2\}$ as

$$f(u_i) = i - 1 \quad \text{for } 1 \leq i \leq 4n-1$$

Clearly vertex labels are distinct. Then f induces a function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$ and for any edge $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. The weights of all the edges of the graph G are distinct. Hence f admits square sum labeling. Therefore I_n is a square sum graph.

In this *Figure 6*, represents the switching of any vertex u_2 in the lilly graph $I_2, (n \geq 2)$ satisfies the prime labeling and coloring conditions with respect to the above theorem 3.11.

Example 3.12

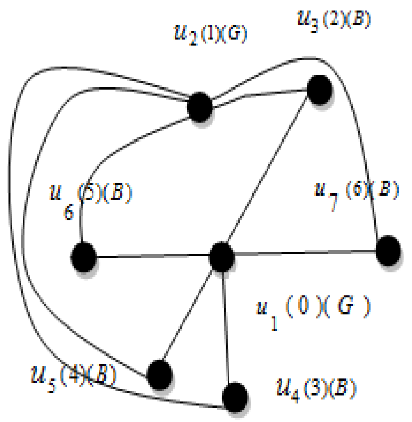

Figure 6 Switching any vertex u_2 in I_2

Theorem 3.13. The switching of an apex vertex u_1 in a lilly graph $I_n, (n \geq 2)$ produces a square sum graph, where *n* is any positive integer.

Proof. Let $G = I_n$ and $u_1, u_2, \ldots, u_{4n-1}$ be the successive vertices of lilly graph $I_n, (n \geq 2)$ and G_u denotes the graph obtained by an apex vertex switching of G with respect to the vertex u_1. It is obvious that $|V(G_u)| = 4n - 1$.Without loss of generality we initiate the labeling from u_1 and proceed in the clock-wise direction. Define a labeling $f: V(G_u) \to \{0, 1,2, \ldots ,4n - 2\}$ as

$$f(u_i) = i - 1 \qquad \text{for } 1 \leq i \leq 4n - 1$$

Clearly vertex labels are distinct. Then f induces a function $f^* : E(G) \to N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$ and for any edge $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. The weights of all the edges of the graph G are distinct. Hence f admits square sum labeling. Therefore I_n is a square sum graph and it is a disconnected graph.

In this *Figure 7*, represents the switching of an apex vertex u_1 in the lilly graph $I_2, (n \geq 2)$ satisfies the prime labeling and coloring conditions with respect to the above theorem 3.13.

Example 3.14

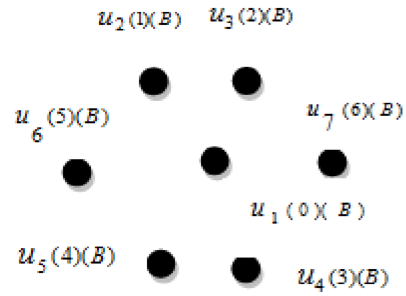

Figure 7 Switching an apex vertex u_1 in I_2.

Applications

Several practical problems in real life situations have motivated the study of labeling of graphs which are required to obey a variety of conditions depending on the structure of graphs. Square sum labeling are applied in the additive number theory, coding theory problems, communication network design, X-ray crystallographic analysis, optimal circuit layout, integral voltage generator, telecommunication.

4.Conclusion

In this paper we proved that the lilly graph I_n, duplication of the lilly graph I_n, fusing of the lilly graph I_n, switching of the lilly graph I_n are square sum graphs and also applied coloring in the lilly graph $I_n$are square sum graphs.

Conflicts of interest

The authors have no conflicts of interest to declare.

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