Research Article

International Journal of Advanced Technology and Engineering Exploration, Vol 4(26)
ISSN (Print): 2394-5443 ISSN (Online): 2394-7454
http://dx.doi.org/10.19101/IJATEE.2017.426001

Optimal PID controller design for level control of three tank system

S. Singh[1*], V. P. Singh[1], R. K. Dohare[2], S. P. Singh[1] and S. K. Jain[1]
Department of Electrical Engineering, National Institute of Technology, Raipur, India[1]
Department of Chemical Engineering, National Institute of Technology, Jaipur, India[2]

Abstract

In this paper, an optimal proportional–integral–derivative controller (PID) controller is designed for level control of three tank system. The integral-square-error (ISE) based approach is used for tuning of PID controller. The ISE of unit step response is minimized for obtaining optimal controller settings. The ISE is constructed in terms of alpha and beta parameters. The teacher learner based optimization (TLBO) technique is used for minimizing the ISE. The TLBO is recently proposed optimization algorithm which is free from algorithm-specific parameters. The results obtained using proposed technique is compared with other well-known methods. It is found that the proposed technique performs much better when compared to others.

Keywords

ISE, Optimization, PID controller, Teacher learner based optimization, Tuning.

1.Introduction

The PID controller is still very much popular in industry because of its simplicity and reliability. Many methods have been proposed in the literature for tuning of parameters of PID controller. One of these methods is Ziegler-Nichols (ZN) settings which is a rule based design criterion [1]. Some other rule based methods are integral of square time weighted error (ISTE), Pessen integral of absolute error (PIAE), Kessler Landau Voda (KLV), some overshoot rule (SO-OV), no overshoot rule (NO-OV), Mantz-Tacconi Ziegler-Nichols (MT-ZN) and refined Ziegler-Nichols (R-ZN) [2].The rule based tuning methods generally provide satisfactory response. The demand of improved dynamic response evolved the various evolutionary computation based tuning techniques. Some of these include Luus-Jaakola optimization procedure for PID controller tuning [3], PID controller tuning based on particle swarm optimization [4], evolutionary computation based PID tuning [5], PID controller tuning based on genetic algorithm [6], PID tuning using soft computing techniques [7], etc. This paper proposes an evolutionary computation based tuning method. The teacher learner based optimization (TLBO) technique is used for obtaining the controller parameters. Being simple and efficient, TLBO is applied for obtaining optimal results for various engineering problems [8-11]. An additional advantage of this algorithm is that it is free from algorithm-specific parameters. The design criterion is ISE of unit step input. The ISE is derived from alpha and beta tables. This method requires only a fixed number of alpha and beta parameters unlike other techniques where the integration up to infinite time is mandatory.

The layout of this brief is as follows. Section 2 describes the three tank system and its model. Section 3 provides the structure of PID controller. The proposed method of tuning is described in section 4. The algorithm utilized to minimize the performance index is discussed in section 5. Section 6 discusses the simulation parameters and quantitative as well as qualitative results obtained. The brief is concluded in section 7.

2.The three tank system

The block diagram shown in *Figure 1* illustrates the closed loop control of three tank system [12].

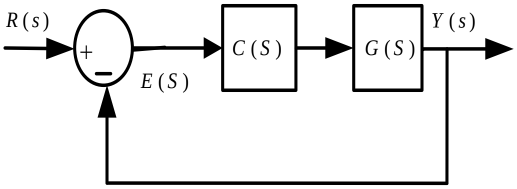

Figure 1 Three tank system

---

[*] Author for correspondence

The transfer function of plant i.e. three tank system is given as

$$G(s)=\frac{K}{(\tau_a s+1)(\tau_b s+1)(\tau_c s+1)} \tag{1}$$

Where, K is gain and τ_a, τ_b and τ_c are time constants of three tanks. The $C(s)$ in block diagram denotes the controller.

3.PID controller

The transfer function of PID controller considered in this work is described as

$$C(s)=K_p\left(1+\frac{1}{T_i s}+T_d\, s\right) \tag{2}$$

Where, K_p, T_i and T_d are controller parameters. The controller settings [2] due to Ziegler-Nichols (ZN), Pessen integral of absolute error (PIAE), some overshoot rule (SO-OV), no overshoot rule (NO-OV) and Mantz-Tacconi Ziegler-Nichols (MT-ZN) criteria are provided in *Table 1*. The parameter K_u denotes the ultimate gain and the parameter T_u represents the period for which ultimate gain occurs.

Table 1 Controller parameters

S.N.	**Rule**	K_p	T_i	T_d
1	ZN	$0.6K_u$	$0.5T_u$	$0.125T_u$
2	PIAE	$0.7K_u$	$0.4T_u$	$0.15T_u$
3	SO-OV	$0.33K_u$	$0.5T_u$	$0.33T_u$
4	NO-OV	$0.2K_u$	$0.5T_u$	$0.33T_u$
5	MT-ZN	$0.6K_u$	$0.5T_u$	$0.125T_u$

4.The proposed method

In proposed method, ISE of unit step response is minimized. The error obtained in s-domain for system given in*Figure 1*.is

$$E(s)=R(s)-Y(s)=\frac{R(s)}{1+G(s)C(s)} \tag{3}$$

where,

$$R(s)=\frac{1}{s} \tag{4}$$

The transfer function of error can be written as

$$E(s)=\frac{b_1 s^{n-1}+\cdots+b_n}{a_0 s^n+a_1 s^{n-1}+\cdots+a_n} \tag{5}$$

The ISE is given as

$$J=\int_{t=0}^{t=\infty} e^2(t)dt \tag{6}$$

which in terms of alpha and beta parameters [13] becomes

$$J=\frac{1}{2}\sum_{i=1}^{n}\frac{\beta_i^2}{\alpha_i} \tag{7}$$

where n is the order of error in s-domain (i.e. $E(s)$ as given by (5))

The alpha and beta parameters are obtained from alpha table and beta table as given in *Table 2* and *Table 3*.

Table 2 Alpha table

	$c_0^0=a_0$	$c_2^0=a_2$	$c_4^0=a_4$	$c_6^0=a_6 \quad \cdots$
	$c_0^1=a_1$	$c_2^1=a_3$	$c_4^1=a_5$	$\cdots$
$\alpha_1=c_0^0/c_0^1$	$c_0^2=c_2^0-\alpha_1 c_2^1$	$c_2^2=c_4^0-\alpha_1 c_4^1$	$c_4^2=c_6^0-\alpha_1 c_6^1$	$\cdots$
$\alpha_2=c_0^1/c_0^2$	$c_0^3=c_2^1-\alpha_2 c_2^2$	$c_2^3=c_4^1-\alpha_2 c_4^2$	$c_4^3=c_6^1-\alpha_2 c_6^2$	$\ddots$
$\alpha_3=c_0^2/c_0^3$	$c_0^4=c_2^2-\alpha_3 c_2^3$	$c_2^4=c_4^2-\alpha_3 c_4^3$	$\vdots$	
$\alpha_4=c_0^3/c_0^4$	$c_0^5=c_2^3-\alpha_4 c_2^4$	$\vdots$		
$\alpha_5=c_0^4/c_0^5$	$\vdots$			
$\vdots$				

Table 3 Beta table

	$d_0^1=b_1$	$d_2^1=b_3$	$d_4^1=b_5$	$d_6^1=b_7 \quad \cdots$
	$d_0^2=b_2$	$d_2^2=b_4$	$d_4^2=b_6$	$\cdots$
$\beta_1=d_0^1/c_0^1$	$d_0^3=d_2^1-\beta_1 c_2^1$	$d_2^3=d_4^1-\beta_1 c_4^1$	$d_4^3=d_6^1-\beta_1 c_6^1$	$\cdots$
$\beta_2=d_0^2/c_0^2$	$d_0^4=d_2^2-\beta_2 c_2^2$	$d_2^4=d_4^2-\beta_2 c_4^2$	$d_4^4=d_6^2-\beta_2 c_6^2$	$\ddots$
$\beta_3=d_0^3/c_0^3$	$d_0^5=d_2^3-\beta_3 c_2^3$	$d_2^5=d_4^3-\beta_3 c_4^3$	$\vdots$	
$\beta_4=d_0^4/c_0^4$	$d_0^6=d_2^4-\beta_4 c_2^4$	$\vdots$		
$\beta_5=d_0^5/c_0^5$	$\vdots$			
$\vdots$				

5.Teacher learner based optimization (TLBO) algorithm

The TLBO algorithm [14] consists two phases: teacher phase and learner phase. In teacher phase, the teacher tries to improve the students' performance to his level. In learner phase, a student interacts to other student of the class to improve his knowledge.

Suppose, there is a total R students in the class and the subjects offered to the students are C. The performance of ith student, $i=1,2,\cdots,R$, in jth subject, $j=1,2,\cdots,C$ is $X_{i,j}$. The teacher tries to improve the performance of students as given below:

$$newX_{i,j} = X_{i,j} + r\left(X_{best,j} - T_f\ \bar{X}_{i,j}\right) \tag{8}$$

where $newX_{i,j}$ is updated performance of the class and r is a random number in the range $(0,1)$. $X_{best,j}$ and $\bar{X}_{i,j}$ denote respectively the performance of best student (i.e. teacher) in the class and mean performance of the class. The parameter T_f is known as teacher factor which represents the teacher capability and is given as

$$T_f = \{1,2\} \tag{9}$$

The teacher factor is determined heuristically and possesses value either 1 or 2 with equal probability. At the end of teacher phase, $newX_{i,j}$ is updated in X if it is a better solution otherwise old performance i.e. $X_{i,j}$ is retained.

In learner phase, each student interact to other to improve his knowledge as given by

$$newX_{i,j} = X_{i,j} + r\left(X_{p,j} - X_{q,j}\right) \tag{10}$$

where p and q are two randomly selected students of the class such that $p \neq q$, $f\left(X_{p,j}\right) < f\left(X_{q,j}\right)$ and $r=(0,1)$.

At the end of learner phase, the solution $newX_{i,j}$ is updated in X if it is better otherwise $X_{i,j}$ is retained.

6.Results and discussion

The parameters [12] of three tank system considered in this work are $K=6$, $\tau_a=2$, $\tau_b=4$ and $\tau_c=6$. For these values, (5) becomes

$$E(s) = \frac{b_1 s^3 + b_2 s^2 + b_3 s + b_4}{a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4} \tag{11}$$

Where,

$$a_0 = T_i\ \tau_a\ \tau_b\ \tau_c \tag{12}$$

$$a_1 = T_i\left(\ \tau_b\ \tau_c + \tau_a\ \tau_c\right) \tag{13}$$

$$a_2 = T_i\left(K\ K_p\ T_d + \tau_c\right) \tag{14}$$

$$a_3 = T_i\left(1 + K\ K_p\right) \tag{15}$$

$$a_4 = K\ K_p \tag{16}$$

$$b_1 = T_i\ \tau_a\ \tau_b\ \tau_c \tag{17}$$

$$b_2 = T_i\left(\ \tau_b\ \tau_c + \tau_a\ \tau_c\right) \tag{18}$$

$$b_3 = T_i\ \tau_c \tag{19}$$

$$b_4 = T_i \tag{20}$$

The alpha and beta tables (*Table 2* and *Table 3*) turn out to be, respectively, *Table 4* and *Table 5.*

Table 4 Alpha table

	$c_0^0 = a_0$	$c_2^0 = a_2$	$c_4^0 = a_4$
	$c_0^1 = a_1$	$c_2^1 = a_3$	$c_4^1 = 0$
$\alpha_1 = c_0^0 / c_0^1$	$c_0^2 = c_2^0 - \alpha_1 c_2^1$	$c_2^2 = c_4^0$	
$\alpha_2 = c_0^1 / c_0^2$	$c_0^3 = c_2^1 - \alpha_2 c_2^2$	$c_2^3 = 0$	
$\alpha_3 = c_0^2 / c_0^3$	$c_0^4 = c_2^2$		
$\alpha_4 = c_0^3 / c_0^4$			

Table 5 Beta table

	$d_0^1 = b_1$	$d_2^1 = b_3$
	$d_0^2 = b_2$	$d_2^2 = b_4$
$\beta_1 = d_0^1 / c_0^1$	$d_0^3 = d_2^1 - \beta_1 c_2^1$	
$\beta_2 = d_0^2 / c_0^2$	$d_0^4 = d_2^2 - \beta_2 c_2^2$	
$\beta_3 = d_0^3 / c_0^3$		
$\beta_4 = d_0^4 / c_0^4$		

The performance index, given by (7) becomes

$$J = \frac{1}{2}\sum_{i=1}^{4}\frac{\beta_i^2}{\alpha_i} \tag{21}$$

$$J = \frac{1}{2}\left\{\begin{array}{l} \dfrac{\left(b_1/a_1\right)^2}{a_0/a_1} + \dfrac{\left(b_2/\left(a_2-\alpha_1 a_3\right)\right)^2}{a_1/\left(a_2-\alpha_1 a_3\right)} + \\ \dfrac{\left(\left(b_3-\beta_1 a_3\right)/\left(a_3-\alpha_2 a_4\right)\right)^2}{\left(a_2-\alpha_1 a_3\right)/\left(a_3-\alpha_2 a_4\right)} + \\ \dfrac{\left(\left(b_4-\beta_2 a_4\right)/a_4\right)^2}{\left(a_3-\alpha_2 a_4\right)/a_4} \end{array}\right\} \tag{22}$$

Minimizing (22) using TLBO, the optimal results obtained are given in *Table 6*. *Table 6* also provides the settings obtained due to ZN, PIAE, SO-OV, NO-OV and MT-ZN rules.

Table 6 Controller parameters

S.N.	Rule	K_p	T_i	T_d
1	Proposed	39.62	948.13	0.23
2	ZN	1	6.28	1.57
3	PIAE	1.17	5.03	1.89
4	SO-OV	0.55	6.28	4.15
5	NO-OV	0.33	6.28	4.15
6	MT-ZN	1	6.28	1.57

Figure 2 provides the step responses obtained due to proposed method and other settings. The settling time and peak over shoot are tabulated in *Table 7*.

From the results obtained, it is clear that the proposed method gives much better controller setting when compared to other methods.

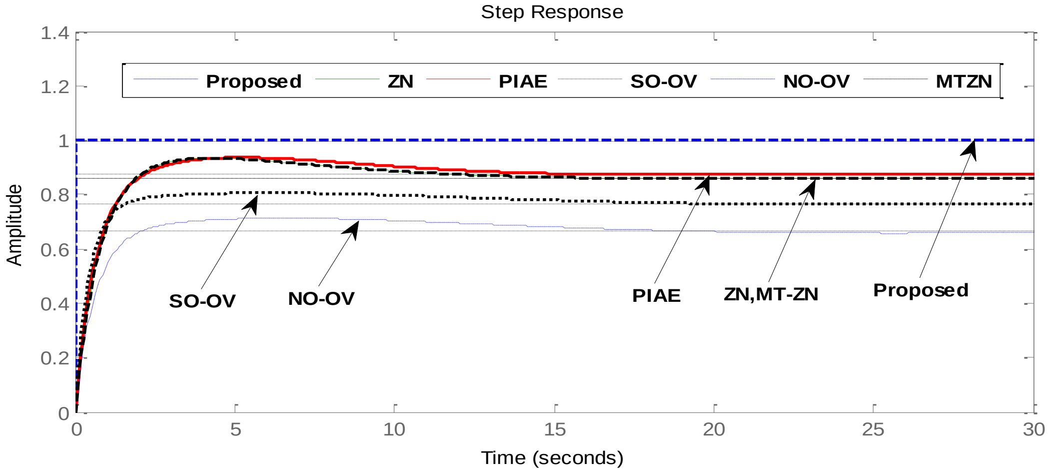

Figure 2 Step response of the system

Table 7 Settling time and peak overshoot

S.N.	Rule	T_s (Sec.)	M_p (%)
1	Proposed	0.003	0
2	ZN	11.90	8.96
3	PIAE	11.50	6.97
4	SO-OV	13.90	5.00
5	NO-OV	15.60	7.29
6	MT-ZN	11.90	8.96

7.Conclusion

In this work, an optimal PID controller is proposed for level control of three tank system. The controller tuning is achieved by minimizing the ISE of unit step response. The ISE is constructed with the help of alpha and beta parameters. The teacher learner based optimization algorithm is utilized for minimizing the ISE. The obtained controller settings show better performance in terms of qualitative and quantitative results.

Acknowledgment

None.

Conflicts of interest

The authors have no conflicts of interest to declare.

References

[1] Ziegler JG, Nichols NB. Optimum settings for automatic controllers. trans. ASME. 1942; 64(11).

[2] McCormack AS, Godfrey KR. Rule-based autotuning based on frequency domain identification. IEEE Transactions on Control Systems Technology. 1998; 6(1):43-61.

[3] Rathore NS, Chauhan DP, Singh VP. Luus-Jaakola optimization procedure for PID controller tuning in reverse osmosis system. In international conference proceedings of IRF, Chennai, India 2015 (pp. 12-15).

[4] Rathore NS, Kundariya N, Narain A. PID controller tuning in reverse osmosis system based on particle swarm optimization. International Journal of Scientific and Research Publications. 2013; 3(6):1-5.

[5] Wadhwani S, Verma V. Evolutionary Computation techniques based optimal PID controller tuning. International Journal of Engineering Trends and Technology. 2013; 4(6):2529-34.

[6] Bindu R, Namboothiripad MK. Tuning of PID controller for DC servo motor using genetic algorithm. International Journal of Emerging Technology and Advanced Engineering. 2012; 2(3):310-4.

[7] Nagaraj B, Subha S, Rampriya B. Tuning algorithms for PID controller using soft computing techniques. International Journal of Computer Science and Network Security. 2008; 8(4):278-81.

[8] Baykasoğlu A, Hamzadayi A, Köse SY. Testing the performance of teaching–learning based optimization (TLBO) algorithm on combinatorial problems: Flow shop and job shop scheduling cases. Information Sciences. 2014; 276:204-18.

[9] Singh M, Panigrahi BK, Abhyankar AR. Optimal coordination of directional over-current relays using teaching learning-based optimization (TLBO) algorithm. International Journal of Electrical Power & Energy Systems. 2013; 50:33-41.

[10] Rao RV, Kalyankar VD. Parameters optimization of advanced machining processes using TLBO algorithm. EPPM, Singapore. 2011; 20:21-31.

[11] Zhile YA, Kang LI, Qun NI, Yusheng XU, Foley A. A self-learning TLBO based dynamic economic/environmental dispatch considering multiple plug-in electric vehicle loads. Journal of Modern Power Systems and Clean Energy. 2014; 2(4):298-307.

[12] Marlin TE. Process control. New York: McGraw-Hill; 2000.

[13] Hutton M, Friedland B. Routh approximations for reducing order of linear, time-invariant systems. IEEE Transactions on Automatic Control. 1975; 20(3):329-37.

[14] Rao RV, Savsani VJ, Vakharia DP. Teaching–learning-based optimization: a novel method for constrained mechanical design optimization problems. Computer-Aided Design. 2011; 43(3):303-15.