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Research Article

Luus-Jaakola based PID controller tuning for double tank system

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Abstract
In this paper, Luus-Jaakola (LJ) optimization based PID controllers are proposed for level control of double tank system. Two controllers namely PI and PID are designed for such level control. The integral-square-error (ISE) of unit step response is considered as design criterion. The ISE constructed requires fixed number of alpha and beta parameters. LJ algorithm is used for minimizing the ISE. The results obtained are compared for three controllers. The proposed scheme shows excellent results in terms of time domain specifications.

Keywords
ISE, Luus-jaakola algorithm, Optimization, PID controller, Tuning.

1.Introduction
The proportional-integral-derivative (PID) controllers are still attracting the researcher because of its simplicity and reliability. Various methods exist in the literature for tuning the PID controllers. Some rule based methods are Ziegler-Nichols (ZN) settings [1], integral of square time weighted error (ISTE), Pessen integral of absolute error (PIAE), Kessler Landau Voda (KLV), some overshoot rule (SO-OV), no overshoot rule (NO-OV), Mantz-Tacconi Ziegler-Nichols (MT-ZN), refined Ziegler-Nichols (R-ZN) [2], etc. Other methods based on swarm intelligence and evolutionary computing include PID controller tuning based on particle swarm optimization [3, 4], PID controller tuning based on genetic algorithm [5, 6], Luus-Jaakola optimization procedure for PID controller tuning [7], evolutionary computation based PID tuning [8], PID tuning using soft computing techniques [9] and colonial competitive algorithm based PID tuning [10]. The latter category provides better time response as well as time domain specifications when compared to rule based tuning criteria.In this work, a swarm intelligence technique is proposed for tuning of PID controllers for level control of double tank system.The performance index is integral-square-error (ISE) of unit step response.The ISE is derived from alpha and beta parameters. The algorithm due to Luus and Jaakola [11] is used for minimizing the performance index.

The layout of this paper is as follows. Section 2 describes the PID controllers. Section 3 provides the double tank system and its model. Luus-Jaakola (LJ) optimization based tuning is described in section 4. The details of LJ optimization are given in section 5. Section 6 provides the simulation parameters and discusses the quantitative and qualitative results obtained. The paper is concluded in section 7.

2.The controllers
The transfer function of proportional-integral-derivative (PID) controller in terms of proportional gain, integral time constant and derivative time constant is given as

$$C(s) = K_p\left(1+\frac{1}{T_i s}+T_d\, s\right) \quad (1)$$

where, K_p, T_i and T_d are, respectively, the proportional gain, integral time constant and derivative time constant. The PI controller is given as

$$C(s) = K_p\left(1+\frac{1}{T_i s}\right) \quad (2)$$

3.Double tank system
The closed loop control of double tank system [12] is given in *Figure 1*. The transfer functions respectively, represents the transfer functions of controller and plant.

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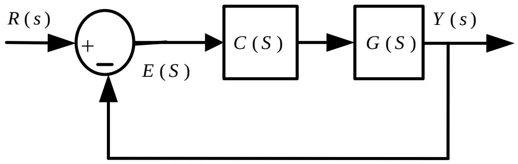

Figure 1 Double tank system

The transfer function double tank system is

$$G(s) = \frac{K}{(\tau_a s+1)(\tau_b s+1)} \tag{3}$$

where, K is the overall gain and τ_a and τ_b are time constants of cascaded two tanks.

4.The proposed approach

The integral-square-error (ISE) of unit step response is considered as design criterion in this work.
The ISE is given as

$$J = \int_{t=0}^{t=\infty} e^2(t)dt \tag{4}$$

where

$$e(t) = r(t) - y(t) \tag{5}$$

In (5), $r(t)$ and $y(t)$ denote, respectively, the desired input and actual output. In this work, desired input is assumed as unit step. Hence,

$$R(s) = \frac{1}{s} \tag{6}$$

The ISE given in (4) can be written in terms of alpha and beta parameters [13] as given by

$$J = \frac{1}{2}\sum_{i=1}^{n} \frac{\beta_i^2}{\alpha_i} \tag{7}$$

Where n is the order of error, $e(t)$, in s-domain. The error in s-domain for system given in *Figure 1* is

$$E(s) = R(s) - Y(s) = \frac{R(s)}{1+G(s)C(s)} \tag{8}$$

The transfer function of error, in general, can be written as

$$E(s) = \frac{v_1 s^{n-1} + \cdots + v_n}{u_0 s^n + u_1 s^{n-1} + \cdots + u_n} \tag{9}$$

The alpha and beta parameters derived from alpha and beta tables are given in *Table 1* and *Table 2*.

Table 1 Alpha table

	$c_0^0 = u_0$	$c_2^0 = u_2$	$c_4^0 = u_4$	$c_6^0 = u_6$	$\cdots$
	$c_0^1 = u_1$	$c_2^1 = u_3$	$c_4^1 = u_5$	$\cdots$	
$\alpha_1 = c_0^0 / c_0^1$	$c_0^2 = c_2^0 - \alpha_1 c_2^1$	$c_2^2 = c_4^0 - \alpha_1 c_4^1$	$c_4^2 = c_6^0 - \alpha_1 c_6^1$	$\cdots$	
$\alpha_2 = c_0^1 / c_0^2$	$c_0^3 = c_2^1 - \alpha_2 c_2^2$	$c_2^3 = c_4^1 - \alpha_2 c_4^2$	$c_4^3 = c_6^1 - \alpha_2 c_6^2$	$\ddots$	
$\alpha_3 = c_0^2 / c_0^3$	$c_0^4 = c_2^2 - \alpha_3 c_2^3$	$c_2^4 = c_4^2 - \alpha_3 c_4^3$	$\vdots$		
$\alpha_4 = c_0^3 / c_0^4$	$c_0^5 = c_2^3 - \alpha_4 c_2^4$	$\vdots$			
$\alpha_5 = c_0^4 / c_0^5$	$\vdots$				
$\vdots$					

Table 2 Beta table

	$d_0^1 = v_1$	$d_2^1 = v_3$	$d_4^1 = v_5$	$d_6^1 = v_7$	$\cdots$
	$d_0^2 = v_2$	$d_2^2 = v_4$	$d_4^2 = v_6$	$\cdots$	
$\beta_1 = d_0^1 / c_0^1$	$d_0^3 = d_2^1 - \beta_1 c_2^1$	$d_2^3 = d_4^1 - \beta_1 c_4^1$	$d_4^3 = d_6^1 - \beta_1 c_6^1$	$\cdots$	
$\beta_2 = d_0^2 / c_0^2$	$d_0^4 = d_2^2 - \beta_2 c_2^2$	$d_2^4 = d_4^2 - \beta_2 c_4^2$	$d_4^4 = d_6^2 - \beta_2 c_6^2$	$\ddots$	
$\beta_3 = d_0^3 / c_0^3$	$d_0^5 = d_2^3 - \beta_3 c_2^3$	$d_2^5 = d_4^3 - \beta_3 c_4^3$	$\vdots$		
$\beta_4 = d_0^4 / c_0^4$	$d_0^6 = d_2^4 - \beta_4 c_2^4$	$\vdots$			
$\beta_5 = d_0^5 / c_0^5$	$\vdots$				
$\vdots$					

5.Luus-Jaakola (LJ) optimization

Luus-Jaakola[11] optimization procedure is simple and efficient optimization technique. The detailed steps are as follow:

1. Choose the initial value of decision variables and region sizes. Let the initial values are X_j^0 with $j = 1, 2, \cdots, C$ where C is the number of decision variables. Suppose the initial region sizes are given as r_j for $j = 1, 2, \cdots, C$.
2. Generate population by

$$X_{i,j} = X_j^0 + \alpha_i r_j \tag{10}$$

Where $\alpha_i = [-0.5, 0.5]$ and $i = 1, 2, \cdots, R$. The parameter R denotes the population size.

3. Check the feasibility of constraint for all the candidates generated by (10). Discard the candidates which are not feasible.
4. Obtain the performance index for all feasible points. Choose the best candidate from population. Suppose the best candidate is $X_{\text{best},j}$. Replace the X_j^0 by $X_{\text{best},j}$ and update the region vector r_j by βr_j where $\beta = [0.9, 0.99]$.
5. Repeat step 2 to step 4 until desired performance index meets.

6. Results and discussion

The parameters [12] of double tank system considered in this work are as follows

$$K = 6,\ \tau_a = 2 \text{ and } \tau_b = 4 \tag{11}$$

PID Controller:

For controller configuration given in (1), error given by (9) becomes

$$E(s) = \frac{v_1 s^2 + v_2 s + v_3}{u_0 s^3 + u_1 s^2 + u_2 s + u_3} \tag{12}$$

where,

$$u_0 = T_i\ \tau_a\ \tau_b \tag{13}$$

$$u_1 = T_i\left(\ \tau_a + \tau_b + K K_p T_d\right) \tag{14}$$

$$u_2 = T_i\left(1 + K\ K_p\right) \tag{15}$$

$$u_3 = K\ K_p \tag{16}$$

$$v_1 = T_i\ \tau_a\ \tau_b \tag{17}$$

$$v_2 = T_i\left(\ \tau_a + \tau_b\right) \tag{18}$$

$$v_3 = T_i \tag{19}$$

The alpha and beta tables (*Table 1* and *Table 2*) modifies to *Table 3* and *Table 4*, respectively for this system.

Table 3 Alpha table

	$u_0^0 = u_0$	$u_2^0 = u_2$
	$u_0^1 = u_1$	$u_2^1 = u_3$
$\alpha_1 = u_0^0 / u_0^1$	$u_0^2 = u_2^0 - \alpha_1 u_2^1$	$u_2^2 = 0$
$\alpha_2 = u_0^1 / u_0^2$	$u_0^3 = u_2^1$	
$\alpha_3 = u_0^2 / u_0^3$		

Table 4 Beta table

	$v_0^1 = v_1$	$v_2^1 = v_3$
	$v_0^2 = v_2$	$v_2^2 = 0$
$\beta_1 = v_0^1 / u_0^1$	$v_0^3 = v_2^1 - \beta_1 u_2^1$	
$\beta_2 = v_0^2 / u_0^2$		
$\beta_3 = v_0^3 / u_0^3$		

The performance index, given by (7) turns out to be

$$J = \frac{1}{2}\sum_{i=1}^{3}\frac{\beta_i^2}{\alpha_i} \tag{20}$$

$$J = \frac{1}{2}\left\{\frac{v_1^2}{u_0 u_1} + \frac{v_2^2}{u_1 u_2 - u_0 u_3} + \frac{\left(u_1 v_3 - v_1 u_3\right)^2}{u_1\left(u_1 u_2 - u_0 u_3\right) u_3}\right\} \tag{21}$$

The optimal values of controller parameters are $K_p = 999.78$, $T_i = 419.68$ and $K_p = 534.82$ when (21) is minimized by LJ. *Figure 2* shows the unit step response obtained for proposed settings. The time domain specifications i.e. settling time and peak over shoot are, respectively, 9.76e-6 seconds and 0%.

PI Controller:

For controller configuration given in (2), (9) becomes

$$E(s) = \frac{v_1 s^2 + v_2 s + v_3}{u_0 s^3 + u_1 s^2 + u_2 s + u_3} \tag{22}$$

where,

$$u_0 = T_i\ \tau_a\ \tau_b \tag{23}$$

$$u_1 = T_i\left(\ \tau_a + \tau_b\right) \tag{24}$$

$$u_2 = T_i\left(1 + K\ K_p\right) \tag{25}$$

$$u_3 = K\ K_p \tag{26}$$

$$v_1 = T_i\ \tau_a\ \tau_b \tag{27}$$

$$v_2 = T_i\left(\ \tau_a + \tau_b\right) \tag{28}$$

$$v_3 = T_i \tag{29}$$

Table 1 and *Table 2* modify to *Table 5* and *Table 6*, respectively.

Table 5 Alpha table

	$u_0^0 = u_0$	$u_2^0 = u_2$
	$u_0^1 = u_1$	$u_2^1 = u_3$
$\alpha_1 = u_0^0 / u_0^1$	$u_0^2 = u_2^0 - \alpha_1 u_2^1$	$u_2^2 = 0$
$\alpha_2 = u_0^1 / u_0^2$	$u_0^3 = u_2^1$	
$\alpha_3 = u_0^2 / u_0^3$		

Table 6 Beta table

	$v_0^1 = v_1$	$v_2^1 = v_3$
	$v_0^2 = v_2$	$v_2^2 = 0$
$\beta_1 = v_0^1 / u_0^1$	$v_0^3 = v_2^1 - \beta_1 u_2^1$	
$\beta_2 = v_0^2 / u_0^2$		
$\beta_3 = v_0^3 / u_0^3$		

The performance index, given by (7) becomes

$$J = \frac{1}{2}\sum_{i=1}^{3}\frac{\beta_i^2}{\alpha_i} \tag{30}$$

$$J = \frac{1}{2}\left\{\frac{v_1^2}{u_0 u_1} + \frac{v_2^2}{u_1 u_2 - u_0 u_3} + \frac{(u_1 v_3 - v_1 u_3)^2}{u_1 (u_1 u_2 - u_0 u_3) u_3}\right\} \tag{31}$$

Minimizing (31) by LJ, the controller parameters obtained are $K_p = 612.73$ and $T_i = 999.98$. The unit step response is also shown in Fig. 2. The settling time and peak over shoot for LJ based PI controller are 10.4 seconds and 94.6%.

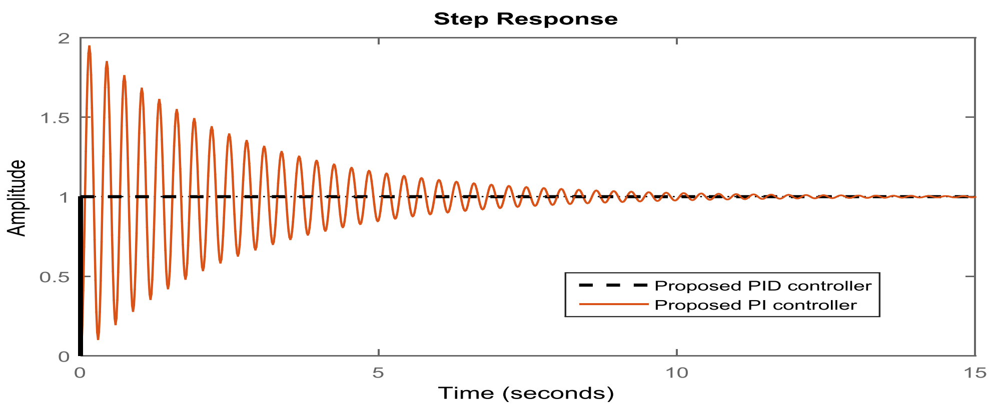

Figure 2 Step response of the system

From the *Figure 2*, it can be concluded that the step responses of PI and PID controllers are excellent in terms of time domain specifications. Hence, it is clear that LJ based tuning of PI and PID controllers provides satisfactory results.

7. Conclusion

PID controllers tuning based on LJ optimization technique is suggested in this work. PI and PID controllers are designed for level control of double tank system. The integral-square-error (ISE) is considered the design criterion. LJ based tuning provides excellent results in terms of time domain specifications.

Acknowledgment

None.

Conflicts of interest

The authors have no conflicts of interest to declare.

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