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Research Article

Tracking control of parallel manipulator with 3-DOF

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Abstract
A parallel three degrees of freedom (3-DOF) manipulator known as Maryland manipulator is considered in this paper. The manipulator model considered in this paper is more practical because the offset lengths are not taken zero. The trajectory tracking control of the Maryland manipulator is done using linear–quadratic regulator (LQR) based proportional–integral–derivative (PID) controller. Three sequential trajectories are used to provide complete dynamic analysis of the manipulator. The manipulator considered here is more practical because the offset lengths are not taken zero unlike previous research articles. The motivation behind using PID controller is its simplicity and effectiveness. The manipulator is a highly coupled and nonlinear system so the correct determination of control signal is very important to achieve good tracking performance. The process for selecting the PID parameters is discussed in detail and the simulations results are used to show the efficacy of the controller.

Keywords
Manipulators, Parallel robots, Inverse kinematics, PID, Robot control.

1.Introduction
A manipulator is fundamentally a reprogrammable, multifunctional mechanical arm intended to move parts, tools, and specialized device through preprogrammed motions for the performance of various tasks. With the advances in technology over last decades Parallel manipulator became more popular for industrial purpose. Parallel manipulators are used for variety of tasks such as flight simulator [1], medical tools [2-4], pick and place operations. The kinematics of parallel manipulator differs from serial manipulator.

Parallel manipulator consists of two platforms one of which is fixed and another is moving platform. These two platforms are connected to each other with the help of limbs. These limbs may consist of revolute or translational joints. Each limb is driven individually. Such type of structure results in closed kinematic mechanism. The kinematic analysis of parallel manipulator is difficult and it has been focus of many researchers [5-8]. The trajectory control of parallel mechanism like this present unique challenges because the dynamics of parallel manipulator are highly coupled and nonlinear. The precision depends on the accuracy of each actuator as well as on the synchronization of the actuators.

Over last few decades many new algorithm are proposed for tracking control but none of them match with PID controller in terms of simplicity and ease of implementation. Zeigler-Nichols proposed an experimental method for tuning of PID controller; though this method has some limitations still it is widely used. In [9] tuning of PID controller is done which is quite similar to ZN method. A fair analysis is done by [10] between different methods of PID controller tuning. In [11], tuning method of PID controller had proposed using gain and phase margin specifications. In [12] authors have discussed a number of PID tuning methods. In [13] author has discussed PID tuning methods categorically.

In [14] had given a complete overview of modern PID tuning methods, different software packages of PID tuner and patents on PID tuning. In [15] authors have given a method for pole placements using PID controllers. In [16] authors proposed a method for tuning of fuzzy based PID controller. In [17] authors have presented an approach for genetically tuning of PID controller. In last few decades researchers have focused on optimal control theory and formulated the well-known optimal state feedback controller known as linear quadratic regulator, this approach reduces the deviation in state trajectories of a system maintaining minimum control effort. A very detailed study of optimal control is presented in [18]. To get the optimal control signal algebraic Riccati equation

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is solved to obtain the transformation matrix P between State and co-state equations of system. The weighting matrices play a crucial role in the solution of control signal. For different weighting matrices the LQR will provide different control signal. In [19] authors proposed a method for tuning of PID controller using LQR approach.

The contribution of this paper is to determine the control action with the help PID controller. The precise and exact control action has great significance for trajectory tracking control of parallel manipulator which presents exclusive challenges.

This paper is organized as follows. System description of Maryland manipulator is presented in section 2. Section 3 presents the parallel manipulator mechanism and motor dynamic model developed by Tsai and Stamper. In section 4 the control strategy is explained in detail. Simulation results of trajectory tracking are shown in section 5 followed by the conclusion in section 6.

2.System description

The manipulator presented here is a 3-DOF parallel manipulator developed by Tsai and Stamper [21] shown in *Figure 1*. The complete schematic diagram of Maryland manipulator is shown in *Figure 2*. It consists of fixed base and a moving platform and three similar limbs which connects the moving platform to the fixed base. Each limb contains two links which are input link and a four bar parallelogram passive link.

The platforms and links are connected to each other through the help of revolute joints. Four parallelogram structures also contain four revolute joints. The presented manipulator has only three degree of freedom since it has three input links which can drive the moving platform effectively in three directions which can be given as X, Y and Z in Cartesian coordinate system.

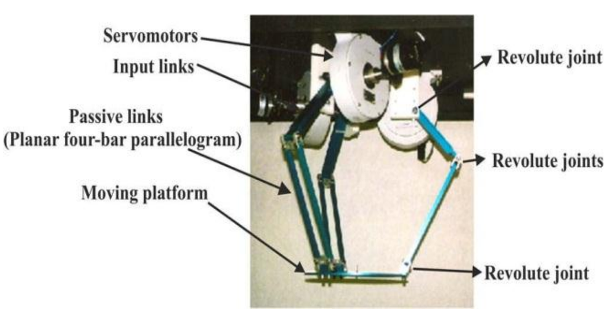

Figure 1 University of Maryland manipulator

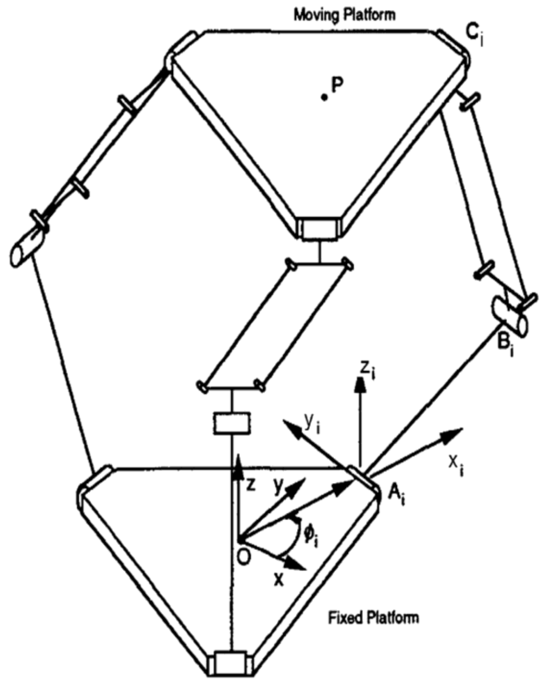

Figure 2 Schematic diagram of manipulator

3.Mathematical model of Maryland manipulator

A complete description of joint angles is given in *Figure 3*. The manipulator parameters are tabulated in *Table 1*. The head of fixed platform is represented by point $A_i(i$ = 1, 2, 3). Let the reference coordinate system be (x, y, z), for the purpose of examination, has its origin at the center of the base platform O and the $x - y$ axes lies on the base platform and the z-axis is perpendicular to the platform. The center P of the moving platform coincides with the origin of the reference frame. The coordinate system attached to fixed platform A_i is (x_i, y_i, z_i), such that the axis x_i is lengthways the stretched line of OA_i, axis y_i is lengthways the revolute joint axis at A_i. The manipulator parameters such as $\theta_{1i}, \theta_{2i}, \theta_{3i}$ can be computed using inverse kinematic problem. Inverse kinematic problem refers to finding the joint variables corresponding to the desired trajectory of the moving platform given in task space using kinematic equation. The inverse kinematic problem can be solved with the help of following set of equations (1) and (2).

A special case has been considered for manipulator by taking the offset length zero [20]. In this paper the similar tracking problem is solved without taking the offset length zero.

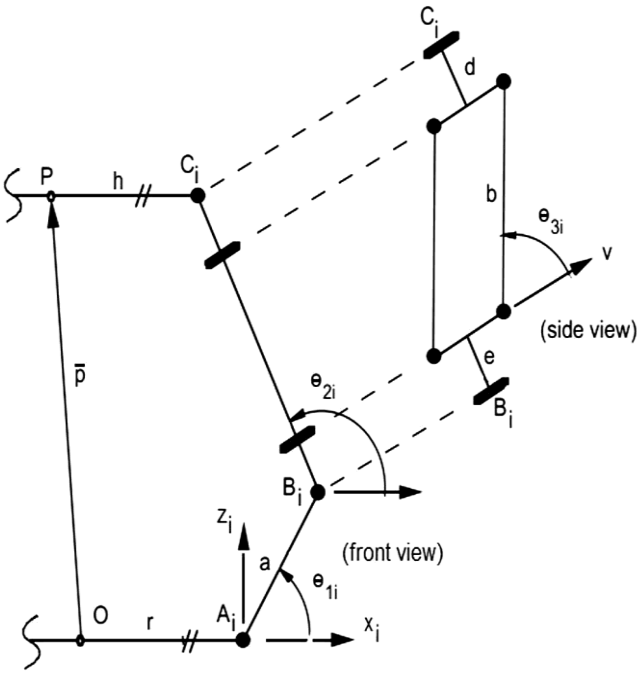

Figure 3 Description of joint angle

$$\begin{bmatrix} a\cos(\theta_{1i}) + \left(d+e+b\sin(\theta_{3i})\right)\cos(\theta_{1i}+\theta_{2i}) \\ b\cos(\theta_{3i}) \\ a\sin(\theta_{1i}) + \left(d+e+b\sin(\theta_{3i})\right)\sin(\theta_{1i}+\theta_{2i}) \end{bmatrix} = \begin{bmatrix} c_{xi} \\ c_{yi} \\ c_{zi} \end{bmatrix} \quad (1)$$

where

$$\begin{bmatrix} c_{xi} \\ c_{yi} \\ c_{zi} \end{bmatrix} = \begin{bmatrix} \cos(\emptyset_i) & \sin(\emptyset_i) & 0 \\ -\sin(\emptyset_i) & \cos(\emptyset_i) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} + \begin{bmatrix} h-r \\ 0 \\ 0 \end{bmatrix} \quad (2)$$

Equation (2) represents the position of point C_i with respect to coordinate frame (x_i, y_i, z_i), and p_x, p_y, p_z are the desired coordinate of the moving platform with in the workspace of manipulator.

By solving the second row of (1) two solution of θ_{3i} can be obtained as given in (3)

$$\theta_{3i} = \pm cos^{-1}\left(\frac{c_{yi}}{b}\right) \quad (3)$$

Similarly θ_{2i} can be found by using the (4)

$$\theta_{2i} = \pm cos^{-1}(k) \quad (4)$$

where

$k = (c_{xi}{}^2 + c_{yi}{}^2 + c_{zi}{}^2 - a^2 - b^2\cos(\theta_{3i}) - (d + e + b\sin(\theta_{3i})))/(2a(d + e + b\sin(\theta_{3i})))$

Corresponding to each solution of θ_{2i},θ_{3i}, (2) yields a unique solution for θ_{1i} which can be given by (5)

$$\theta_{1i} = \pm cos^{-1}\left(\frac{c_{zi}k_2 + c_{xi}k_1}{k_1{}^2 + k_2{}^2}\right) \quad (5)$$

where

$k_1 = a + b\sin(\theta_{3i})\cos(\theta_{2i}) + (d+e)\cos(\theta_{2i})$

$k_2 = b\sin(\theta_{3i})\sin(\theta_{2i}) + (d+e)\sin(\theta_{2i})$

Table 1 Parameters for manipulator

Symbol	Description	Value
θ_{1i}	Actuator angle measured from x_i axis to A_iB_i	Find inverse kinematic (rad)
θ_{2i}	Passive angle is defined from the extended line of A_iB_i to the line defined by the intersection of the plane of the parallelogram and the x_i-z_i plane	Find inverse kinematic (rad)
θ_{3i}	Passive angle is measured from the y_i direction to B_iC_i	Find inverse kinematic (rad)
$\emptyset_{1i}$	Angle is measured from the x-axis to the x_i-axis and is a constant parameter of the manipulator design	[0,120,240] (deg)
a	Length of links A_iB_i	203.2 mm
b	Length of links B_iC_i	254.0 mm
e, d	Small offset length of in parallelogram	0.0159 mm
r	Radius of fixed platform	127.0 mm
h	Radius of moving platform	127.0 mm
m_a	Mass of input link	0.184 kg
m_b	Mass of the one of the two connecting road	0.085 kg
m_p	Mass of the moving platform	0.413 kg
I_m	Axial moment of inertia of the rotor mounted on *i*th limb	0.00434 N.m.s^2

Lagrangian approach is used here to obtain the equation of motion which describes the actuating torques. The motivation behind using the Lagrangian approach is the complex kinematics of manipulator. We take the assumption, as proposed by Tsai [21], that the mass of each rod, m_b, is evenly divide and concentrated at two end points B_i, C_i, to simplify the analysis.

As proposed by [20] the total kinetic energy of system is:

$$T = \frac{1}{2}m_p(\dot{p_x}^2 + \dot{p_y}^2 + \dot{p_z}^2) + \frac{1}{2}(I_m + \frac{1}{3}m_a a^2)\dot{\theta_{1i}}^2 + \frac{1}{2}m_b(\dot{p_x}^2 + \dot{p_y}^2 + \dot{p_z}^2) + \frac{1}{2}m_b a^2\dot{\theta_{1i}}^2 \quad (6)$$

The total potential energy of the system is given as

$$P = m_p g_c p_z + \frac{1}{2}m_a g_c a\sin(\theta_{1i}) + m_b g_c(p_z + a\sin(\theta_{1i})) \quad (7)$$

The Lagrangian equation of motion is given as

$$\frac{d}{dt}\left(\frac{\partial L}{\partial q_i}\right) - \frac{\partial L}{\partial q_i} + \Sigma_{k=1}^{3}\lambda_k\frac{\partial \Gamma_i}{\partial q_i} = Q_i \quad \text{for } i = 1 \text{ to } 6 \quad (8)$$

where
j = is the constraint index,
i = is the generalized coordinate index,
k = is the number of constrained function,
L = is the Lagrange function, where $L = T - P$,
T = is the total kinetic energy of system,
P = total potential energy of system,
q_i = is the i^{th}generalized coordinate,
Q_i = is a generalized external force,
∂q_i = is the i^{th} generalized coordinate,
λ = is the Lagrange multiplier,
Γ_i = is a constraint equation,
The constraint equation used to find the solution of Lagrangian equation of motion using the fact that the distance between joints B and C is constantly same as the length of the upper linking arm, b; that is

$$\Gamma_k = B_iC_i^2 - b^2 = 0$$

The desired trajectory tracking control can be accomplished by applying the torques, computed with the help of above given set of equations, to the input link of the each limb. The torque required is generated with help of three direct current motors (DC Motor).

The general block diagram structure for independent joint control is shown in *Figure 4*.

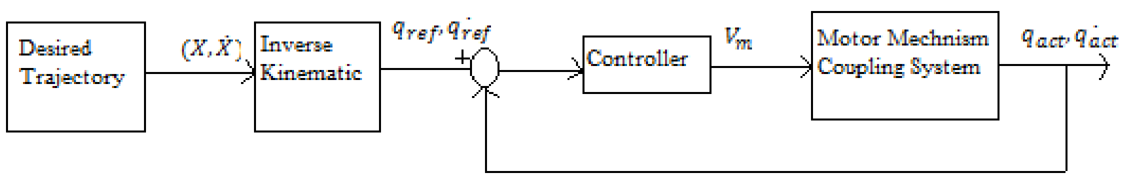

Figure 4 Block scheme of general independent joint control.

A DC motor is used for generating the required torque. The output of motor is proportionate to the input voltage applied to DC motor by the controller. The controller produces the input voltage such that the error i.e. $e = q_{ref} - q_{act}$ tend to zero. The type of controller applied is PID controller whose tuning approaches are discussed in detail in subsequent section.
q_{ref} = desired input link angle in radian
q_{act} = actual input link angle in radian

4.Control strategy

The control strategy used for trajectory tracking control is LQR based PID controller. The weight matrices of LQR is selected by trial and error method. In the presents approach we assume that the error, the integral of error and derivative of it are the state variables and designs the optimal state feedback controller gains as the PID controller parameters. In *Figure 5* if the system is excited with an external input say r(t) to have a control signal u(t) and the system output y(t), then the system state variable defined as:

$$x_1 = \int e(t)dt,\ x_2 = e(t),\ x_3 = \frac{de(t)}{dt} \quad (9)$$

From the *Figure 2*, it is clear that

$$\frac{Y(s)}{U(s)} = \frac{K}{S^2+2\zeta\omega_n S+\omega n^2} = -\frac{E(s)}{U(s)} \quad (10)$$

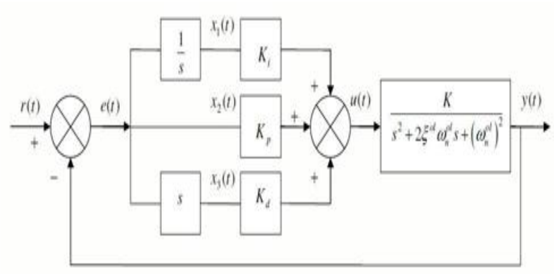

Figure 5 LQR formulation of PID controller

The tracking problem is converted to regulator problem by making r(t) = 0. So when there is no change in the set point equation (10) can be written as

$$[s^2 + 2\zeta\omega_n s + \omega_n^2]E(s) = -KU(S) \quad (11)$$

$$\ddot{e} + 2\zeta\omega_n\dot{e} + \omega_n^2 e = -ku \quad (12)$$

Using equation (9) and (12) we can write

$$\dot{x_3} + 2\zeta\omega_n x_3 + \omega_n^2 x_2 = -ku \quad (13)$$

Using equation (9) and (13) the state space model is given as:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -\omega_n^2 & -2\zeta\omega_n\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ -k\end{bmatrix}u \quad (14)$$

From given state space model the system matrices given as:

$$A = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -\omega_n^2 & -2\zeta\omega_n\end{bmatrix}, \quad B = \begin{bmatrix}0\\ 0\\ -k\end{bmatrix}$$

The quadratic cost function to be minimized for the LQR formulation of the system given by following equation:

$$\int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \quad (15)$$

The minimization of cost function gives the state feedback control signal as:

$$u(t) = -R^{-1}B^T P\, x(t) = -k\, x(t) \quad (16)$$

Where P is the symmetric positive definite matrix obtained by solving the continuous algebraic Riccati Equation (CARE) given by

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (17)$$

Here, the weight matrix Q is a positive symmetric semi-definite and the weight factor R is positive integer. The selection of Q and R matrices is done by Trial and Error, generally R is kept fixed and Q matrix is varied.

P and Q are given by:

$$P = \begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{12} & P_{22} & P_{23}\\ P_{13} & P_{23} & P_{33}\end{bmatrix}, \quad Q = \begin{bmatrix}Q_1 & 0 & 0\\ 0 & Q_2 & 0\\ 0 & 0 & Q_3\end{bmatrix} \quad (18)$$

Now let the unique solution of the CARE (17) be P, so the control signal corresponding to the solution of Algebraic Riccati Equation is given by:

$$u = -Kx(t) = -R^{-1}[0 \quad 0 \quad -k]\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{12} & P_{22} & P_{23}\\ P_{13} & P_{23} & P_{33}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$u = R^{-1}k[P_{13} \quad P_{23} \quad P_{33}]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

$$u = [K_i \quad K_p \quad K_d]\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} \quad (19)$$

$$u = K_i\int e(t)dt + K_p e(t)dt + K_d\frac{de(t)}{dt} \quad (20)$$ (12)

The above given formulation shows that with the appropriate choice of Q and R matrices a PID controller can be tuned and so tuned PID controller will provide minimum state deviation with minimum control effort and better steady state response.

5.Simulation

The proposed methods for controlling the manipulator (14) are tested using MATLAB simulations. The required torque is produced using DC motor. To validate the presented control approaches we have used three sequential trajectories which are given as follows:

$$\left.\begin{aligned}p_x &= 0\\ p_y &= 0\\ p_z &= 0.35 + \frac{t^2}{100}\end{aligned}\right\} \text{if } t \le 5$$

$$\left.\begin{aligned}p_x &= 0.06\cos(t)\\ p_y &= 0.06\sin(t)\\ p_z &= 0.35\end{aligned}\right\} \text{if } 5 < t \le 10 \quad (15)$$

$$\left.\begin{aligned}p_x &= 0\\ p_y &= 0\\ p_z &= 0.35 - \frac{t^2}{100}\end{aligned}\right\} \text{if } t > 10 \quad (16) \quad ()$$

The graphical representation of the each three axis with respect to time is given in *Figure 6*.

The graphical representation of input link angle (radian), obtained solving the inverse kinematic problem, of first limb is shown in *Figure 7*. (17)

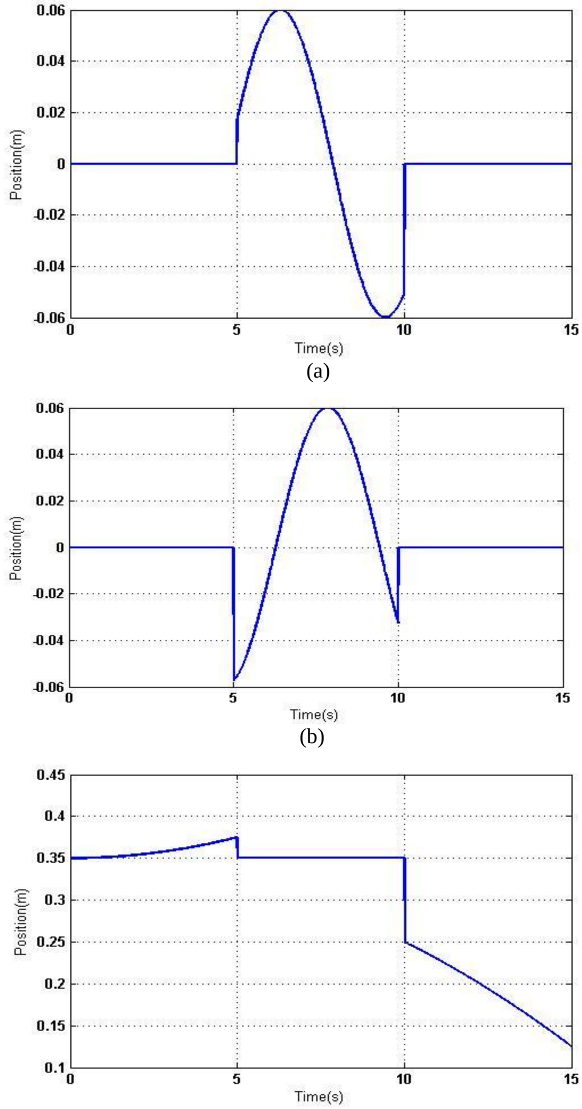

Figure 6 Graphical representation of three trajectories for three axis. (a) x-axis (b) y-axis (c) z-axis

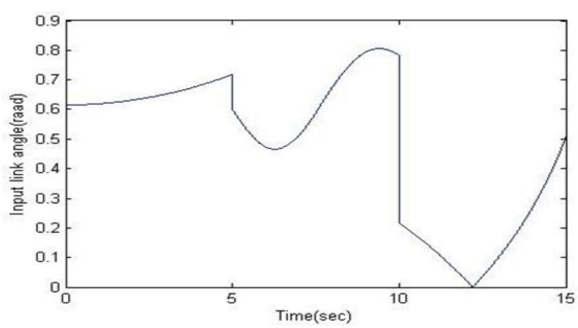

Figure 7 Desired input link angle corresponding to first limb

The motor can be modeled as simply relating torque equation with the voltage equation. The gain of the motor under such case is taken as 0.99 N.m/volt. The generated torque is applied to single link dynamic model which can be obtained by linearization of the dynamics of manipulator. This model is based on the assumption that each link can be modeled separately; it implies that the motion of the rest of the manipulator does not influence the input link that is being modeled. The single link dynamic model obtained is given as follows:

$$\frac{\theta_{11}}{\tau_{11}} = \frac{1}{\alpha s^2 + c_d s + \beta \sin(\pi/4)}$$

Where

$$\alpha = I_m + \frac{1}{3} m_a a^2 + m_{eq} a^2$$

$$\beta = ag\left(\frac{1}{2} m_a + m_{eq}\right)$$

$$m_{eq} = 2m_b + \frac{1}{3} m_c$$

After implementing the general block diagram scheme as shown in *Figure 4*, using the above discussed control approaches, the results obtained are as following. The weight matrices selected are as following.

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R = [0.01]$$

The closed loop response of the system is shown in the *Figure 8.* The results are obtained by using linear simulation in MATLAB. It can be seen that there are no overshoots in case of LQR based PID controller.

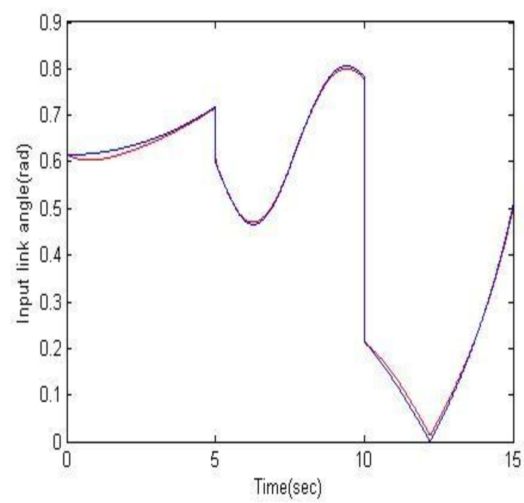

Figure 8 Obtained and desired input link angle for LQR based PID controller

6.Conclusion

In this paper a 3-DOF manipulator is used which also knows as Maryland manipulator. LQR based PID control approach is implemented on this manipulator and results are analysed using MATLAB simulations. First, the system description and dynamics were introduced using Tsai and Stamper method. In the next section a DC motor model was introduced which act as an actuator for the given system. In the next section tuning methods for PID controller is discussed. The control approaches than were implemented using MATLAB software to validate them.

Acknowledgment

None.

Conflicts of interest

The authors have no conflicts of interest to declare.

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