Switching of Picosecond Pulses under Third Order Dispersion Effects in a Non Linear Fiber Coupler

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Abstract

In this paper, we numerically study the propagation and switching of the picosecond pulses under normal and anomalous dispersion regimes. If a strong optical pulse is given as input to a single core of two core fiber coupler under anomalous dispersion regime, then as the pulse in one core decreases its amplitude; correspondingly the pulse in other core increases its amplitude. The power exchange is possible only when the guides are made of material with third order optical susceptibility. The non linear susceptibility gives rise to self trapping of power in the original guide. Hence after a coupling length the output power changes from 0% to 100% in the original guide.

Keywords
Optical Soliton, Split Step Fourier Transform (SSFM), Third order Dispersion, Transmittance, Coupling Coefficient.

1. Introduction

The non-linear couplers provide the excellent soliton switching for optical materials. The energy of the couplers is interchanged between the guiding medium in proper manner [1]. The propagation of pulse in the optical core of the fiber which is governed by Non-Linear Schrödinger Equation (NLSE) is solved with the help of Split Step Fourier Transform Method [3]. The Transmission coefficient [4] is determined to check how efficiently coupling takes place between the two cores. When the switching device is a two core fiber coupler, optical power from the core 1 is periodically switched to core 2 depending on the minimum distance necessary to transfer 100% of power which is called as coupling length L and it is given L=π/2K and K is the coupling coefficient by [5]. We study the effects of third order dispersion over soliton switching in nonlinear directional couplers [7].

2. Modelling of single mode fiber

The pulse propagated in optical fiber coupler involves the range of short width pulses from 10ns to 10fs. When these pulses propagate inside the fiber, the dispersion and nonlinear parameter affects their shape and spectrum. The propagation of the optical pulse for single mode fiber is governed by the basic equation which is given by

$$\frac{\partial A}{\partial z} + i\beta_2 \frac{\partial^2 A}{\partial t^2} - i\gamma |A|^2 A + \frac{\alpha}{2} A = 0$$  \hspace{1cm} (1)

Where, A is the pulse Amplitude, β₂ is the Group Velocity Dispersion for second order, which is the linear parameter and γ is the non linear parameter. It is tedious to solve the above partial differential equation by analytical method and so we make use of the numerical method. The commonly used numerical method in solving NLSE is Split Step Fourier Transform method because of its simple nature in all aspects [6]. It is a Pseudo-spectral method which provides the solution for Non-Linear Schrödinger Equation. The naming convention of SSFM is because of two reasons. They are

(i) The solution is computed separately for linear and non-linear in terms of small steps.

(ii) Since the linear step is computed in the frequency domain and the nonlinear step in the time domain, it is essential to take Fourier transform back and forth.

Hence, propagation of optical pulses from z to z + h is splitted as z to z + h/2 which is nonlinear part and z + h/2 to h which is the linear part and equation (1) is written accordingly as

$$\frac{\partial A}{\partial z} = (\hat{D} + \hat{N})A$$  \hspace{1cm} (2)

Where,

$$\hat{D} = -\frac{i\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - \frac{\alpha}{2}$$  \hspace{1cm} (3)
\[\hat{N} = i\gamma |A|^2 \quad (4)\]

A different procedure is adopted to acquire the accurate results in SSFM over one segment from \(z\) to \(z+h\). In this procedure equation (1) is replaced by

\[A(z+h,T) \approx \exp\left(\frac{h}{2} \hat{D} \right) \exp\left(\int_{z}^{z+h} \hat{N}(z')dz' \right) \exp\left(\frac{h}{2} \hat{D} \right) A(z,T) \quad (5)\]

This is called Symmetrized Split Step Fourier Method. The propagation of soliton in the fiber coupler is affected by the sign of GVD parameter. If \(\beta_2 > 0\), it is called normal dispersion regime in which the low frequency component of the optical pulse travel faster than the high frequency component of the same pulse and hence we get pulse broadening at the output end. If \(\beta_2 < 0\), it is called Anomalous dispersion regime in which the low frequency component of the optical pulse travel slower than the high frequency component of the same pulse. Also when length of the fiber increases, the dispersion increases. Hence the propagation of pulse inside a optical fiber depends on fiber length, non linear and linear parameter of the pulse [8]. It can be clearly seen from the following results in Figure 1 and 2.

When we introduce third order GVD parameter \(\beta_3\), there exists chirping. When \(\beta_3\) is positive, chirping appears near the trailing edge of the pulse and when \(\beta_3\) is negative chirping appears near the leading edge of the pulse. Optical pulse with time dependency in phase shift is called chirp. Due to this chirp, the pulse gets broadened at the fiber output in leading or trailing edge depending on positive or negative value of third order GVD parameter [7] which is clearly seen in Figure 3 and 4.

3. Modelling of coupled mode fiber

A non-linear coupler consists of two single mode waveguides. Coupling is the process of transferring of signals which is introduced in the channel 1(direct channel) to the channel 2 (cross channel) for the period of coupling length. In coupled mode theory, switching plays a vital role. Compared to continuous wave, soliton has more advantages since soliton wave can maintain their shape during the pulse propagation. The main advantage of soliton is the effect of the dispersion and the non-linearity getting cancelled with each other at any point in the fiber [2].

The soliton propagation occurs only in negative dispersion (i.e.) anomalous dispersion. It is seen from the figure 5, 6 and 7 that amplitude of the soliton pulse diminishes as it approaches the edge of the first core whereas in the second core the soliton becomes sharper and it regains its amplitude slowly as it approaches to the end.

The non-linear Schrödinger equation for coupled fiber is given by

\[\frac{\partial u}{\partial z} = i\beta_2 \frac{\partial^2 u}{\partial t^2} + \beta_3 \frac{\partial^3 u}{\partial t^3} + i\gamma |u|^2 u + iKv \quad (6)\]

\[\frac{\partial v}{\partial z} = i\beta_2 \frac{\partial^2 v}{\partial t^2} + \beta_3 \frac{\partial^3 v}{\partial t^3} + i\gamma |v|^2 v + iKu \quad (7)\]

Where \(u\) and \(v\) are varying magnitudes in first and second core, \(\beta_2\) is taken as negative and \(K\) is the coupling coefficient between two fibers. The linear and non-linear part of equation (5) is

\[\frac{1}{2} \frac{\partial u}{\partial z} = i\beta_2 \frac{\partial^2 u}{\partial t^2} + \beta_3 \frac{\partial^3 u}{\partial t^3} + iKv \quad (8)\]

\[\frac{1}{2} \frac{\partial u}{\partial z} = i\gamma |u|^2 u \quad (9)\]

Similarly, for equation (6) it is

\[\frac{1}{2} \frac{\partial v}{\partial z} = i\beta_2 \frac{\partial^2 v}{\partial t^2} + \beta_3 \frac{\partial^3 v}{\partial t^3} + iKu \quad (10)\]

\[\frac{1}{2} \frac{\partial v}{\partial z} = i\gamma |v|^2 v \quad (11)\]

These equations are solved by the same procedure which is followed for solving single NLSE using Split Step Fourier Transform. The pulse profile and the evolution of pulses for various design parameters are mentioned in the figure 5, 6 and 7.
Figure 1: Pulse profile and Evolution of pulse for design parameters $\beta_2=10 \text{ ps}^2 \text{ km}^{-1}$, $\gamma=0.001 \text{ W}^{-1} \text{ m}^{-1}$ and $t_0=100 \text{ ps}$

Figure 2: Pulse profile and Evolution of pulse for design parameters $\beta_2=-10 \text{ ps}^2 \text{ km}^{-1}$, $\gamma=0.001 \text{ W}^{-1} \text{ m}^{-1}$ and $t_0=100 \text{ ps}$

Figure 3: Pulse profile and Evolution of pulse for design parameters $\beta_2=-10 \text{ ps}^2 \text{ km}^{-1}$, $\beta_3=1 \text{ ps}^3 \text{ m}^{-1}$, $\gamma=0.001 \text{ W}^{-1} \text{ m}^{-1}$ and $t_0=100 \text{ ps}$

Figure 4: Pulse profile and Evolution of pulse for design parameters $\beta_2=-10 \text{ ps}^2 \text{ km}^{-1}$, $\beta_3=-1 \text{ ps}^3 \text{ m}^{-1}$, $\gamma=0.001 \text{ W}^{-1} \text{ m}^{-1}$ and $t_0=100 \text{ ps}$
Figure 5: Pulse profile and Evolution of pulses for u, v and Transmission characteristics for u, v for design parameters $\beta_2=10$ ps$^2$/km, $\beta_3=1$ ps/m, $\gamma=0.001$ W/m, $t_0=100$ ps and coupling coefficient $K=0.157$

Figure 6: Pulse profile and Evolution of pulses for u, v and Transmission characteristics for u, v for design parameters $\beta_2=10$ ps$^2$/km, $\beta_3=1$ ps/m, $\gamma=0.001$ W/m, $t_0=100$ ps and coupling coefficient $K=0.0785$
Figure 7: Pulse profile and Evolution of pulses for \( u, v \) and Transmission characteristics for \( u, v \) for design parameters \( \beta_2 = -10 \text{ ps}^2 \text{ km}^{-1}, \beta_3 = -1 \text{ps}^3 \text{ m}^{-1}, \gamma = 0.001 \text{W}^{-1}\text{m}^{-1}, t_0 = 100 \text{ ps} \) and coupling coefficient \( K = 0.0523 \)

4. Results and Discussions

The Split Step Fourier Transform method is applied for the above single mode and coupled mode equations to determine the propagation of pulse in the fibers. Let consider the initial conditions applied to single NLSE equation to be,

\[ u(0,t) = P \text{sech}(t) \]  
(12)

Here \( P \) is the input peak power. We can neglect the initial chirp \( C \) and the loss parameter \( \alpha \) which is negligible in ideal soliton.

The parameters involved in the experimental set up are \( P = 0.001 \text{watts}, \alpha = 0, \beta_2 = 10 \text{ps}^2 \text{km}^{-1}, \gamma = 0.001 \text{W}^{-1}\text{m}^{-1}, t_0 = 100 \text{ps} \). Solving the equation by changing the magnitude of the parameter \( \beta_2 \), we get two different results which are shown in Figure 1 and 2. Consider the case (i) \( \beta_2 = 10 \text{ps}^2 \text{km}^{-1} \) (ii) \( \beta_2 = -10 \text{ps}^2 \text{km}^{-1} \). It is noted that the pulse undergoes dispersion only if \( \beta_2 \) is positive (i.e. under normal dispersion regime). When we introduce third order GVD parameter \( \beta_3 \) in equation (1), the results says that when \( \beta_3 = +1 \text{ps}^3\text{km}^{-1} \), chirping appear near the trailing edge of the pulse and when \( \beta_3 = -1 \text{ps}^3\text{m}^{-1} \), chirping appear near the leading edge of the pulse as shown in figure 3 and 4.

In coupled mode equations, consider the initial conditions to be

\[ u(0,t) = P \text{sech}(t) \]  
(12)

\[ v(0,t) = 0 \]  
(13)

The half-beat length of the coupling fiber is given by \( L = \pi/2K \). The parameters which are taken to analyze the switching characteristics are \( P = 0.001 \text{watts}, \alpha = 0, \beta_2 = 10 \text{ps}^2 \text{km}^{-1}, \beta_3 = -1 \text{ps}^3\text{m}^{-1}, \gamma = 0.001 \text{W}^{-1}\text{m}^{-1}, t_0 = 100 \text{ps} \) and coupling coefficient \( K \). The coupling coefficient determines the coupling length of the fiber.

The transmission coefficient \( T \) is given by the formula [4]

\[
T = \frac{\int_{-\infty}^{\infty} |u(z,t)|^2 \ dt}{\int_{-\infty}^{\infty} (|u(z,t)|^2 + |v(z,t)|^2) \ dt}
\]

(15)  

The characteristics of transmission coefficient of the second core pulse \( V \) with respect to variations in the coupling coefficient \( K \) is plotted and is given by the figure 8. It is seen that as coupling coefficient value increases, the transmission coefficient increases linearly.

Table 1: Variation of Transmission coefficient \( T \) in core 2 for various values of coupling coefficient \( K \).

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<thead>
<tr>
<th>COUPLING COEFFICIENT K</th>
<th>TRANSMISSION COEFFICIENT FOR V</th>
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We concern about the propagation of pulses in single mode and coupled mode fiber under normal and anomalous dispersion regimes with the effect of second and third order dispersion. We have applied Split step Fourier Transform method to find the solution of coupled nonlinear Schrödinger equation. The Transmittance curve of the dual core fiber is plotted that shows the switching characteristics. The transmission characteristics can be altered by changing the value of coupling coefficient. Hence the coupling coefficient determines the coupling length of the fiber. Currently, We have been working on the effect of Self steepening and Raman effect in switching of picoseconds pulses along with third order dispersion effects.

References


